

# Chapter Vector Algebra



## Topic-1: Algebra of Vectors, Linear Dependence & Independence of Vectors, Vector Inequality



### 1 MCQs with One Correct Answer

- Let P, Q, R and S be the points on the plane with position vectors  $-2\hat{i} - \hat{j}$ ,  $4\hat{i}$ ,  $3\hat{i} + 3\hat{j}$  and  $-3\hat{i} + 2\hat{j}$  respectively. The quadrilateral PQRS must be a [2010]
  - parallelogram, which is neither a rhombus nor a rectangle
  - square
  - rectangle, but not a square
  - rhombus, but not a square
- Let  $\alpha$ ,  $\beta$ ,  $\gamma$  be distinct real numbers. The points with position vectors  $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ ,  $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$ ,  $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$ 
  - are collinear [1994]
  - form an equilateral triangle
  - form a scalene triangle
  - form a right angled triangle
- The points with position vectors  $60\hat{i} + 3\hat{j}$ ,  $40\hat{i} - 8\hat{j}$ ,  $a\hat{i} - 52\hat{j}$  are collinear if [1983 - 1 Mark]
  - $a = -40$
  - $a = 40$

(c)  $a = 20$

(d) none of these



### 2 Integer Value Answer/ Non-Negative Integer

- Consider the set of eight vectors  $V = \{a\hat{i} + b\hat{j} + c\hat{k} : a, b, c \in \{-1, 1\}\}$ . Three non-coplanar vectors can be chosen from V in  $2^p$  ways. Then p is [Adv. 2013]



### 5 True / False

- The points with position vectors  $a + b$ ,  $a - b$ , and  $a + kb$  are collinear for all real values of k. [1984 - 1 Mark]



### 6 MCQs with One or More than One Correct Answer

- If  $a = i + j + k$ ,  $\vec{b} = 4i + 3j + 4k$  and  $c = i + \alpha j + \beta k$  are linearly dependent vectors and  $|c| = \sqrt{3}$ , then [1998 - 2 Marks]
  - $\alpha = 1, \beta = -1$
  - $\alpha = 1, \beta = \pm 1$
  - $\alpha = -1, \beta = \pm 1$
  - $\alpha = \pm 1, \beta = 1$





7 Match the Following

7. Match the following :

**Column I**

- (A) In a triangle  $\Delta XYZ$ , let  $a, b$ , and  $c$  be the lengths of the sides opposite to the angles  $X, Y$  and  $Z$ , respectively. If  $2(a^2 - b^2) = c^2$  and  $\lambda = \frac{\sin(X - Y)}{\sin Z}$ , then possible values of  $n$  for which  $\cos(n\pi\lambda) = 0$  is (are)
- (B) In a triangle  $\Delta XYZ$ , let  $a, b$  and  $c$  be the lengths of the sides opposite to the angles  $X, Y$ , and  $Z$  respectively. If  $1 + \cos 2X - 2\cos 2Y = 2 \sin X \sin Y$ , then possible value (s) of  $\frac{a}{b}$  is (are)
- (C) In  $R^2$ , let  $\sqrt{3}\hat{i} + \hat{j}, \hat{i} + \sqrt{3}\hat{j}$  and  $\beta\hat{i} + (1 - \beta)\hat{j}$  be the position vectors of  $X, Y$  and  $Z$  with respect to the origin  $O$ , respectively. If the distance of  $Z$  from the bisector of the acute angle of  $\overline{OX}$  with  $\overline{OY}$  is  $\frac{3}{\sqrt{2}}$ , then possible value(s) of  $|\beta|$  is (are)
- (D) Suppose that  $F(\alpha)$  denotes the area of the region bounded by  $x = 0, x = 2, y^2 = 4x$  and  $y = |\alpha x - 1| + |\alpha x - 2| + \alpha x$ , where  $\alpha \in \{0, 1\}$ . Then the value(s) of  $F(\alpha) + \frac{8}{3}\sqrt{2}$ , when  $\alpha = 0$  and  $\alpha = 1$ , is (are)

**Column II**  
(p) 1

(q) 2

(r) 3

(s) 5

(t) 6

[Adv. 2015]

8. Match the following :

**Column I**

- (A) In  $R^2$ , if the magnitude of the projection vector of the vector  $\alpha\hat{i} + \beta\hat{j}$  on  $\sqrt{3}\hat{i} + \hat{j}$  is  $\sqrt{3}$  and if  $\alpha = 2 + \sqrt{3}\beta$ , then possible value of  $|\alpha|$  is/are
- (B) Let  $a$  and  $b$  be real numbers such that the function  $f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ bx + a^2, & x \geq 1 \end{cases}$  if differentiable for all  $x \in R$ . Then possible value of  $a$  is (are)
- (C) Let  $\omega \neq 1$  be a complex cube root of unity. If  $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$ , then possible value (s) of  $n$  is (are)
- (D) Let the harmonic mean of two positive real numbers  $a$  and  $b$  be 4. If  $q$  is a positive real number such that  $a, 5, q, b$  is an arithmetic progression, then the value(s) of  $|q - a|$  is (are)

**Column II**  
(p) 1

(q) 2

(r) 3

(s) 4

(t) 5

[Adv. 2015]



10 Subjective Problems

9. Show, by vector methods, that the angular bisectors of a triangle are concurrent and find an expression for the position vector of the point of concurrency in terms of the position vectors of the vertices. [2001 - 5 Marks]
10. Prove, by vector methods or otherwise, that the point of intersection of the diagonals of a trapezium lies on the line passing through the mid-points of the parallel sides. (You may assume that the trapezium is not a parallelogram.) [1998 - 8 Marks]
11. In a triangle  $ABC$ ,  $D$  and  $E$  are points on  $BC$  and  $AC$  respectively, such that  $BD = 2DC$  and  $AE = 3EC$ . Let  $P$  be the point of intersection of  $AD$  and  $BE$ . Find  $BP/PE$  using vector methods. [1993 - 5 Marks]
12. In a triangle  $OAB$ ,  $E$  is the midpoint of  $BO$  and  $D$  is a point on  $AB$  such that  $AD : DB = 2 : 1$ . If  $OD$  and  $AE$  intersect at  $P$ , determine the ratio  $OP : PD$  using vector methods. [1989 - 4 Marks]
13. Let  $OACB$  be a parallelogram with  $O$  at the origin and  $OC$  a diagonal. Let  $D$  be the midpoint of  $OA$ . Using vector methods prove that  $BD$  and  $CO$  intersect in the same ratio. Determine this ratio. [1988 - 3 Marks]
14. A vector  $\vec{A}$  has components  $A_1, A_2, A_3$  in a right-handed rectangular Cartesian coordinate system  $oxyz$ . The coordinate system is rotated about the  $x$ -axis through an angle  $\frac{\pi}{2}$ . Find the components of  $A$  in the new coordinate system, in terms of  $A_1, A_2, A_3$ . 1983 - 2 Marks]





Topic-2: Scalar or Dot Product of two Vectors



1 MCQs with One Correct Answer

1. Let  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{c} = \hat{i} - \hat{j} - \hat{k}$  be three vectors. A vector  $\vec{v}$  in the plane of  $\vec{a}$  and  $\vec{b}$ , whose projection on  $\vec{c}$  is  $\frac{1}{\sqrt{3}}$ , is given by [2011]

- (a)  $\hat{i} - 3\hat{j} + 3\hat{k}$  (b)  $-3\hat{i} - 3\hat{j} - \hat{k}$   
 (c)  $3\hat{i} - \hat{j} + 3\hat{k}$  (d)  $\hat{i} + 3\hat{j} - 3\hat{k}$

2. Let two non-collinear unit vectors  $\hat{a}$  and  $\hat{b}$  form an acute angle. A point  $P$  moves so that at any time  $t$  the position vector  $\vec{OP}$  (where  $O$  is the origin) is given by  $\hat{a} \cos t + \hat{b} \sin t$ . When  $P$  is farthest from origin  $O$ , let  $M$  be the length of  $\vec{OP}$  and  $\hat{u}$  be the unit vector along  $\vec{OP}$ . Then, [2008]

- (a)  $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$  and  $M = (1 + \hat{a} \cdot \hat{b})^{1/2}$   
 (b)  $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$  and  $M = (1 + \hat{a} \cdot \hat{b})^{1/2}$   
 (c)  $\hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$  and  $M = (1 + 2\hat{a} \cdot \hat{b})^{1/2}$   
 (d)  $\hat{u} = \frac{\hat{a} - \hat{b}}{|\hat{a} - \hat{b}|}$  and  $M = (1 + 2\hat{a} \cdot \hat{b})^{1/2}$

3. If  $\vec{a}, \vec{b}, \vec{c}$  are three non-zero, non-coplanar vectors and

$$\vec{b}_1 = \vec{b} - \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a}, \vec{b}_2 = \vec{b} + \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|^2} \vec{a},$$

$$\vec{c}_1 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} + \frac{\vec{b} \cdot \vec{c}}{|\vec{c}|^2} \vec{b}_1, \vec{c}_2 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{b}_1 \cdot \vec{c}}{|\vec{b}_1|^2} \vec{b}_1,$$

$$\vec{c}_3 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} + \frac{\vec{b} \cdot \vec{c}}{|\vec{c}|^2} \vec{b}_1, \vec{c}_4 = \vec{c} - \frac{\vec{c} \cdot \vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{b}_1 \cdot \vec{c}}{|\vec{b}_1|^2} \vec{b}_1,$$

then the set of orthogonal vectors is [2005S]

- (a)  $(\vec{a}, \vec{b}_1, \vec{c}_3)$  (b)  $(\vec{a}, \vec{b}_1, \vec{c}_2)$   
 (c)  $(\vec{a}, \vec{b}_1, \vec{c}_1)$  (d)  $(\vec{a}, \vec{b}_2, \vec{c}_2)$

4. If  $\vec{a}$  and  $\vec{b}$  are two unit vectors such that  $\vec{a} + 2\vec{b}$  and  $5\vec{a} - 4\vec{b}$  are perpendicular to each other then the angle between  $\vec{a}$  and  $\vec{b}$  is [2002S]

- (a)  $45^\circ$  (b)  $60^\circ$   
 (c)  $\cos^{-1}\left(\frac{1}{3}\right)$  (d)  $\cos^{-1}\left(\frac{2}{7}\right)$

5. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are unit vectors, then

$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$  does NOT exceed [2001S]  
 (a) 4 (b) 9 (c) 8 (d) 6

6. Let  $\vec{u}, \vec{v}$  and  $\vec{w}$  be vectors such that  $\vec{u} + \vec{v} + \vec{w} = 0$ . If  $|\vec{u}| = 3, |\vec{v}| = 4$  and  $|\vec{w}| = 5$ , then  $\vec{u} \cdot \vec{v} + \vec{v} \cdot \vec{w} + \vec{w} \cdot \vec{u}$  is [1995S]

- (a) 47 (b) -25 (c) 0 (d) 25

7. Let  $\vec{p}$  and  $\vec{q}$  be the position vectors of  $P$  and  $Q$  respectively, with respect to  $O$  and  $|\vec{p}| = p, |\vec{q}| = q$ . The points  $R$  and  $S$  divide  $PQ$  internally and externally in the ratio 2 : 3 respectively. If  $OR$  and  $OS$  are perpendicular then [1994]

- (a)  $9q^2 = 4p^2$  (b)  $4p^2 = 9q^2$  (c)  $9p = 4q$  (d)  $4p = 9q$



2 Integer Value Answer/ Non-Negative Integer

8. Let  $\vec{p} = 2\hat{i} + \hat{j} + 3\hat{k}$  and  $\vec{q} = \hat{i} - \hat{j} + \hat{k}$ . If for some real numbers  $\alpha, \beta$  and  $\gamma$ , we have

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(2\vec{p} + \vec{q}) + \beta(\vec{p} - 2\vec{q}) + \gamma(\vec{p} \times \vec{q}),$$

then the value of  $\gamma$  is [Adv. 2024]

9. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are unit vectors satisfying [2012]

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9, \text{ then } |2\vec{a} + 5\vec{b} + 5\vec{c}| \text{ is}$$



4 Fill in the Blanks

10. The components of a vector  $\vec{a}$  along and perpendicular to a non-zero vector  $\vec{b}$  are.....and.....respectively. [1988 - 2 Marks]

11. Let  $\vec{b} = 4\hat{i} + 3\hat{j}$  and  $\vec{c}$  be two vectors perpendicular to each other in the  $xy$ -plane. All vectors in the same plane having projections 1 and 2 along  $\vec{b}$  and  $\vec{c}$ , respectively, are given by..... [1987 - 2 Marks]

12.  $A, B, C$  and  $D$ , are four points in a plane with position vectors  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  respectively such that

$$(\vec{a} - \vec{d})(\vec{b} - \vec{c}) = (\vec{b} - \vec{d})(\vec{c} - \vec{a}) = 0 \quad [1984 - 2 Marks]$$

The point  $D$ , then, is the..... of the triangle  $ABC$ .

13. Let  $\vec{A}, \vec{B}, \vec{C}$  be vectors of length 3, 4, 5 respectively. Let  $\vec{A}$  be perpendicular to  $\vec{B} + \vec{C}$ ,  $\vec{B}$  to  $\vec{C} + \vec{A}$  and  $\vec{C}$  to  $\vec{A} + \vec{B}$ . Then the length of vector  $\vec{A} + \vec{B} + \vec{C}$  is ..... [1981 - 2 Marks]

**6 MCQs with One or More than One Correct Answer**

14. The vector  $\frac{1}{3}(2\hat{i} - 2\hat{j} + \hat{k})$  is [1994]  
 (a) a unit vector  
 (b) makes an angle  $\frac{\pi}{3}$  with the vector  $(2\hat{i} - 4\hat{j} + 3\hat{k})$

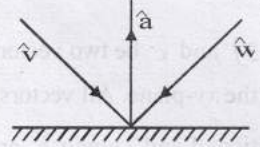
**7 Match the Following**

16. Match the statements given in Column-I with the values given in Column-II. [2011]

Column-I	Column-II
(A) If $\vec{a} = \hat{j} + \sqrt{3}\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then the internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\frac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x) dx = a^2 - b^2$ , then the value of $f\left(\frac{\pi}{6}\right)$ is	(q) $\frac{2\pi}{3}$
(C) The value of $\frac{\pi^2}{\ln 3} \int_{7/6}^{5/6} \sec(\pi x) dx$ is	(r) $\frac{\pi}{3}$
(D) The maximum value of $\left  \text{Arg}\left(\frac{1}{1-z}\right) \right $ for $ z  = 1, z \neq 1$ is given by	(s) $\pi$
	(t) $\frac{\pi}{2}$

**10 Subjective Problems**

17. If the incident ray on a surface is along the unit vector  $\hat{v}$ , the reflected ray is along the unit vector  $\hat{w}$  and the normal is along unit vector  $\hat{a}$  outwards. Express  $\hat{w}$  in terms of  $\hat{a}$  and  $\hat{v}$ . [2005 - 4 Marks]



18. Let  $\vec{A}(t) = f_1(t)\hat{i} + f_2(t)\hat{j}$  and  $\vec{B}(t) = g_1(t)\hat{i} + g_2(t)\hat{j}, t \in [0, 1]$ , where  $f_1, f_2, g_1, g_2$  are continuous functions. If  $\vec{A}(t)$  and  $\vec{B}(t)$  are nonzero vectors for all  $t$  and  $\vec{A}(0) = 2\hat{i} + 3\hat{j}, \vec{A}(1)$

(c) parallel to the vector  $\left(-\hat{i} + \hat{j} - \frac{1}{2}\hat{k}\right)$   
 (d) perpendicular to the vector  $3\hat{i} + 2\hat{j} - 2\hat{k}$   
 15. Let  $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - \hat{k}$  and  $\vec{c} = \hat{i} + \hat{j} - 2\hat{k}$  be three vectors. A vector in the plane of  $\vec{b}$  and  $\vec{c}$ , whose projection on  $\vec{a}$  is of magnitude  $\sqrt{2/3}$ , is: [1993 - 2 Marks]  
 (a)  $2\hat{i} + 3\hat{j} - 3\hat{k}$  (b)  $2\hat{i} + 3\hat{j} + 3\hat{k}$   
 (c)  $-2\hat{i} - \hat{j} + 5\hat{k}$  (d)  $2\hat{i} + \hat{j} + 5\hat{k}$

$= 6\hat{i} + 2\hat{j}, \vec{B}(0) = 3\hat{i} + 2\hat{j}$  and  $\vec{B}(1) = 2\hat{i} + 6\hat{j}$ . Then show that  $\vec{A}(t)$  and  $\vec{B}(t)$  are parallel for some  $t$ . [2001 - 5 Marks]  
 19. Find 3-dimensional vectors  $\vec{v}_1, \vec{v}_2, \vec{v}_3$  satisfying  $\vec{v}_1 \cdot \vec{v}_1 = 4, \vec{v}_1 \cdot \vec{v}_2 = -2, \vec{v}_1 \cdot \vec{v}_3 = 6, \vec{v}_2 \cdot \vec{v}_2 = 2, \vec{v}_2 \cdot \vec{v}_3 = -5, \vec{v}_3 \cdot \vec{v}_3 = 29$  [2001 - 5 Marks]  
 20. Determine the value of 'c' so that for all real x, the vector  $cx\hat{i} - 6\hat{j} - 3\hat{k}$  and  $x\hat{i} + 2\hat{j} + 2cx\hat{k}$  make an obtuse angle with each other. [1991 - 4 Marks]  
 21. From a point O inside a triangle ABC, perpendiculars OD, OE, OF are drawn to the sides BC, CA, AB respectively. Prove that the perpendiculars from A, B, C to the sides EF, FD, DE are concurrent. [1978]

Topic-3: Vector or Cross Product of two vectors, Scalar & Vector Triple Product



1 MCQs with One Correct Answer

- Let the position vectors of the points P, Q, R and S be  $\vec{a} = \hat{i} + 2\hat{j} - 5\hat{k}$ ,  $\vec{b} = 3\hat{i} + 6\hat{j} + 3\hat{k}$ ,  $\vec{c} = \frac{17}{5}\hat{i} + \frac{16}{5}\hat{j} + 7\hat{k}$  and  $\vec{d} = 2\hat{i} + \hat{j} + \hat{k}$ , respectively. Then which of the following statements is true? [Adv. 2023]
  - The points P, Q, R and S are NOT coplanar.
  - $\frac{\vec{b} + 2\vec{d}}{3}$  is the position vector of a point which divides PR internally in the ratio 5 : 4.
  - $\frac{\vec{b} + 2\vec{d}}{3}$  is the position vector of a point which divides PR externally in the ratio 5 : 4.
  - The square of the magnitude of the vector  $\vec{b} \times \vec{d}$  is 95.
- If  $\vec{a}$  and  $\vec{b}$  are vectors such that  $|\vec{a} + \vec{b}| = \sqrt{29}$  and  $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$ , then a possible value of  $(\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k})$  is [2012]
  - 0
  - 3
  - 4
  - 8
- Two adjacent sides of a parallelogram ABCD are given by  $\vec{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$  and  $\vec{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$ . The side AD is rotated by an acute angle  $\alpha$  in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle  $\alpha$  is given by [2010]
  - $\frac{8}{9}$
  - $\frac{\sqrt{17}}{9}$
  - $\frac{1}{9}$
  - $\frac{4\sqrt{5}}{9}$
- If  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  are unit vectors such that  $(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$  and  $\vec{a} \cdot \vec{c} = \frac{1}{2}$ , then [2009]
  - $\vec{a}, \vec{b}, \vec{c}$  are non-coplanar
  - $\vec{b}, \vec{c}, \vec{d}$  are non-coplanar
  - $\vec{b}, \vec{d}$  are non-parallel
  - $\vec{a}, \vec{d}$  are parallel and  $\vec{b}, \vec{c}$  are parallel
- The edges of a parallelepiped are of unit length and are parallel to non-coplanar unit vectors  $\hat{a}, \hat{b}, \hat{c}$  such that  $\hat{a} \cdot \hat{b} = \hat{b} \cdot \hat{c} = \hat{c} \cdot \hat{a} = \frac{1}{2}$ . Then, the volume of the parallelepiped is [2008]
  - $\frac{1}{\sqrt{2}}$
  - $\frac{1}{2\sqrt{2}}$
  - $\frac{\sqrt{3}}{2}$
  - $\frac{1}{\sqrt{3}}$
- Let  $\vec{a}, \vec{b}, \vec{c}$  be unit vectors such that  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ . Which one of the following is correct? [2007 -3 marks]
  - $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} = \vec{0}$
  - $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$
  - $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{a} \times \vec{c} = \vec{0}$
  - $\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a}$  are mutually perpendicular
- The number of distinct real values of  $\lambda$ , for which the vectors  $-\lambda^2\hat{i} + \hat{j} + \hat{k}$ ,  $\hat{i} - \lambda^2\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} - \lambda^2\hat{k}$  are coplanar, is [2007 -3 marks]
  - zero
  - one
  - two
  - three
- Let  $\vec{a} = \hat{i} + 2\hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} - \hat{j} + \hat{k}$  and  $\vec{c} = \hat{i} + \hat{j} - \hat{k}$ . A vector in the plane of  $\vec{a}$  and  $\vec{b}$  whose projection on  $\vec{c}$  is  $\frac{1}{\sqrt{3}}$ , is [2006 - 3M, -1]
  - $4\hat{i} - \hat{j} + 4\hat{k}$
  - $3\hat{i} + \hat{j} - 3\hat{k}$
  - $2\hat{i} + \hat{j} - 2\hat{k}$
  - $4\hat{i} + \hat{j} - 4\hat{k}$
- The unit vector which is orthogonal to the vector  $5\hat{i} + 2\hat{j} + 6\hat{k}$  and is coplanar with the vectors  $2\hat{i} + \hat{j} + \hat{k}$  and  $\hat{i} - \hat{j} + \hat{k}$  is [2004S]
  - $\frac{2\hat{i} - 6\hat{j} + \hat{k}}{\sqrt{41}}$
  - $\frac{2\hat{i} - 3\hat{j}}{\sqrt{13}}$
  - $\frac{3\hat{j} - \hat{k}}{\sqrt{10}}$
  - $\frac{4\hat{i} + 3\hat{j} - 3\hat{k}}{\sqrt{34}}$
- If  $\vec{a} = (\hat{i} + \hat{j} + \hat{k})$ ,  $\vec{a} \cdot \vec{b} = 1$  and  $\vec{a} \times \vec{b} = \hat{j} - \hat{k}$ , then  $\vec{b}$  is [2004S]
  - $\hat{i} - \hat{j} + \hat{k}$
  - $2\hat{j} - \hat{k}$
  - $\hat{i}$
  - $2\hat{i}$
- The value of 'a' so that the volume of parallelepiped formed by  $\hat{i} + a\hat{j} + \hat{k}$ ,  $\hat{j} + a\hat{k}$  and  $a\hat{i} + \hat{k}$  becomes minimum is [2003S]
  - 3
  - 3
  - $1/\sqrt{3}$
  - $\sqrt{3}$
- Let  $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{W} = \hat{i} + 3\hat{k}$ . If  $\vec{U}$  is a unit vector, then the maximum value of the scalar triple product  $|\vec{U} \cdot \vec{V} \times \vec{W}|$  is [2002S]
  - 1
  - $\sqrt{10} + \sqrt{6}$
  - $\sqrt{59}$
  - $\sqrt{60}$
- Let  $\vec{a} = \vec{i} - \vec{k}$ ,  $\vec{b} = x\vec{i} + \vec{j} + (1-x)\vec{k}$  and  $\vec{c} = y\vec{i} + x\vec{j} + (1+x-y)\vec{k}$ . Then  $[\vec{a} \vec{b} \vec{c}]$  depends on [2001S]
  - only x
  - only y
  - Neither x Nor y
  - both x and y



14. If  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are unit coplanar vectors, then the scalar triple product  $\left[ 2\vec{a}-\vec{b}, 2\vec{b}-\vec{c}, 2\vec{c}-\vec{a} \right] =$  [2000S]

- (a) 0 (b) 1 (c)  $-\sqrt{3}$  (d)  $\sqrt{3}$

15. Let the vectors  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  be such that

$$\left( \vec{a} \times \vec{b} \right) \times \left( \vec{c} \times \vec{d} \right) = 0.$$

Let  $P_1$  and  $P_2$  be planes determined by the pairs of vectors  $\vec{a}, \vec{b}$  and  $\vec{c}, \vec{d}$  respectively. Then the angle between  $P_1$  and  $P_2$  is [2000S]

- (a) 0 (b)  $\frac{\pi}{4}$  (c)  $\frac{\pi}{3}$  (d)  $\frac{\pi}{2}$

16. If the vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  form the sides  $BC, CA$  and  $AB$  respectively of a triangle  $ABC$ , then [2000S]

(a)  $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = 0$

(b)  $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

(c)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a}$

(d)  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a} = \vec{0}$

17. Let  $a=2i+j+k, b=i+2j-k$  and a unit vector  $c$  be coplanar. If  $c$  is perpendicular to  $a$ , then  $c =$  [1999 - 2 Marks]

(a)  $\frac{1}{\sqrt{2}}(-j+k)$  (b)  $\frac{1}{\sqrt{3}}(-i-j-k)$

(c)  $\frac{1}{\sqrt{5}}(i-2j)$  (d)  $\frac{1}{\sqrt{3}}(i-j-k)$

18. Let  $a = 2i + j - 2k$  and  $b = i + j$ . If  $c$  is a vector such that  $a \cdot c = |c|, |c - a| = 2\sqrt{2}$  and the angle between  $(a \times b)$  and  $c$  is  $30^\circ$ , then  $|(a \times b) \times c| =$  [1999 - 2 Marks]

- (a)  $2\sqrt{3}$  (b)  $3/2$  (c) 2 (d) 3

19. If  $\vec{a}, \vec{b}$  and  $\vec{c}$  are three non coplanar vectors, then [1995S]

$(\vec{a} + \vec{b} + \vec{c}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})]$  equals

- (a) 0 (b)  $[\vec{a} \vec{b} \vec{c}]$   
(c)  $2[\vec{a} \vec{b} \vec{c}]$  (d)  $-[\vec{a} \vec{b} \vec{c}]$

20. If  $\vec{a}, \vec{b}, \vec{c}$  are non coplanar unit vectors such that

$\vec{a} \times (\vec{b} \times \vec{c}) = \frac{(\vec{b} + \vec{c})}{\sqrt{2}}$ , then the angle between  $\vec{a}$  and  $\vec{b}$  is [1995S]

- (a)  $\frac{3\pi}{4}$  (b)  $\frac{\pi}{4}$  (c)  $\pi/2$  (d)  $\pi$

21. Let  $\vec{a} = \hat{i} - \hat{j}, \vec{b} = \hat{j} - \hat{k}, \vec{c} = \hat{k} - \hat{i}$ . If  $\vec{d}$  is a unit vector such that  $\vec{a} \cdot \vec{d} = 0 = [\vec{b} \vec{c} \vec{d}]$ , then  $\vec{d}$  equals [1995S]

(a)  $\pm \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}}$  (b)  $\pm \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

(c)  $\pm \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{3}}$  (d)  $\pm \hat{k}$

22. Let  $a, b, c$  be distinct non-negative numbers. If the vectors  $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}$  and  $c\hat{i} + c\hat{j} + b\hat{k}$  lie in a plane, then  $c$  is

- (a) the Arithmetic Mean of  $a$  and  $b$  [1993 - 1 Marks]  
(b) the Geometric Mean of  $a$  and  $b$   
(c) the harmonic Mean of  $a$  and  $b$   
(d) equal to zero

23. Let  $\vec{a}, \vec{b}, \vec{c}$ , be three non-coplanar vectors and  $\vec{p}, \vec{q}, \vec{r}$ , are vectors defined by the relations  $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a} \vec{b} \vec{c}]}, \vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a} \vec{b} \vec{c}]},$

$\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a} \vec{b} \vec{c}]}$  then the value of the expression

$(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r}$  is equal to [1988 - 2 Marks]

- (a) 0 (b) 1 (c) 2 (d) 3.

24. The volume of the parallelepiped whose sides are given by  $\vec{OA} = 2i - 2j, \vec{OB} = i + j - k, \vec{OC} = 3i - k$ , is [1983 - 1 Mark]


- (a)  $\frac{4}{13}$  (b) 4  
(c)  $\frac{2}{7}$  (d) none of these

25. For non-zero vectors  $\vec{a}, \vec{b}, \vec{c}$ ,  $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}| |\vec{b}| |\vec{c}|$  holds if and only if [1982 - 2 Marks]

- (a)  $\vec{a} \cdot \vec{b} = 0, \vec{b} \cdot \vec{c} = 0$  (b)  $\vec{b} \cdot \vec{c} = 0, \vec{c} \cdot \vec{a} = 0$   
(c)  $\vec{c} \cdot \vec{a} = 0, \vec{a} \cdot \vec{b} = 0$  (d)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

26. The scalar  $\vec{A} \cdot (\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})$  equals : [1981 - 2 Marks]

- (a) 0 (b)  $[\vec{A} \vec{B} \vec{C}] + [\vec{B} \vec{C} \vec{A}]$   
(c)  $[\vec{A} \vec{B} \vec{C}]$  (d) None of these

 2 Integer Value Answer/Non-Negative Integer

27. Let  $\vec{u}, \vec{v}$  and  $\vec{w}$  be vectors in three-dimensional space, where  $\vec{u}$  and  $\vec{v}$  are unit vectors which are not perpendicular to each other and  $\vec{u} \cdot \vec{w} = 1, \vec{v} \cdot \vec{w} = 1, \vec{w} \cdot \vec{w} = 4$ . If the volume of the parallelepiped, whose adjacent sides are represented by the vectors  $\vec{u}, \vec{v}$  and  $\vec{w}$ , is  $\sqrt{2}$ , then the value of  $|3\vec{u} + 5\vec{v}|$  is \_\_\_\_\_. [Adv. 2021]

28. Let  $\vec{a}$  and  $\vec{b}$  be two unit vectors such that  $\vec{a} \cdot \vec{b} = 0$ . For

some  $x, y \in \mathbb{R}$ , let  $\vec{c} = x\vec{a} + y\vec{b} + (\vec{a} \times \vec{b})$ . If  $|\vec{c}| = 2$  and the vector  $\vec{c}$  is inclined at the same angle  $\alpha$  to both  $\vec{a}$  and  $\vec{b}$ , then the value of  $8 \cos^2 \alpha$  is \_\_\_\_\_. [Adv. 2018]

29. Suppose that  $\vec{p}, \vec{q}$  and  $\vec{r}$  are three non-coplanar vectors in  $\mathbb{R}^3$ . Let the components of a vector  $\vec{s}$  along  $\vec{p}, \vec{q}$  and  $\vec{r}$  be 4, 3 and 5, respectively. If the components of this vector  $\vec{s}$  along  $(-\vec{p} + \vec{q} + \vec{r}), (\vec{p} - \vec{q} + \vec{r})$  and  $(-\vec{p} - \vec{q} + \vec{r})$  are  $x, y$  and  $z$ , respectively, then the value of  $2x + y + z$  is \_\_\_\_\_. [Adv. 2015]

30. Let  $\vec{a}, \vec{b}$  and  $\vec{c}$  be three non-coplanar unit vectors such that the angle between every pair of them is  $\frac{\pi}{3}$ . If  $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} = p\vec{a} + q\vec{b} + r\vec{c}$ , where  $p, q$  and  $r$  are scalars, then the value of  $\frac{p^2 + 2q^2 + r^2}{q^2}$  is \_\_\_\_\_. [Adv. 2014]

31. Let  $\vec{a} = -\hat{i} - \hat{k}, \vec{b} = -\hat{i} + \hat{j}$  and  $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$  be three given vectors. If  $\vec{r}$  is a vector such that  $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$  and  $\vec{r} \cdot \vec{a} = 0$ , then the value of  $\vec{r} \cdot \vec{b}$  is \_\_\_\_\_. [2011]

32. If  $\vec{a}$  and  $\vec{b}$  are vectors in space given by  $\vec{a} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$  and  $\vec{b} = \frac{2\hat{i} + \hat{j} + 3\hat{k}}{\sqrt{14}}$ , then find the value of  $(2\vec{a} + \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$ . [2010]

**3 Numeric/ New Stem Based Questions**

33. Let  $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$  be two vectors. Consider a vector  $\vec{c} = \alpha\vec{a} + \beta\vec{b}, \alpha, \beta \in \mathbb{R}$ . If the projection of  $\vec{c}$  on the vector  $(\vec{a} + \vec{b})$  is  $3\sqrt{2}$ , then the minimum value of  $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c}$  equals \_\_\_\_\_. [Adv. 2019]

**4 Fill in the Blanks**

34. Let  $OA = a, OB = 10a + 2b$  and  $OC = b$  where  $O, A$  and  $C$  are non-collinear points. Let  $p$  denote the area of the quadrilateral  $OABC$ , and let  $q$  denote the area of the parallelogram with  $OA$  and  $OC$  as adjacent sides. If  $p = kq$ , then  $k = \dots$ . [1997 - 2 Marks]

35. If  $\vec{b}$  and  $\vec{c}$  are any two non-collinear unit vectors and  $\vec{a}$  is any vector, then  $(\vec{a} \cdot \vec{b})\vec{b} + (\vec{a} \cdot \vec{c})\vec{c} + \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|}(\vec{b} \times \vec{c}) = \dots$ . [1996 - 2 Marks]

36. A unit vector coplanar with  $\vec{i} + \vec{j} + 2\vec{k}$  and  $\vec{i} + 2\vec{j} + \vec{k}$  and perpendicular to  $\vec{i} + \vec{j} + \vec{k}$  is \_\_\_\_\_. [1992 - 2 Marks]

37. If the vectors  $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} + c\hat{k}$  ( $a \neq b \neq c \neq 1$ ) are coplanar, then the value of  $\frac{1}{(1-a)} + \frac{1}{(1-b)} + \frac{1}{(1-c)} = \dots$ . [1987 - 2 Marks]

38. If  $\vec{A} = (1, 1, 1), \vec{C} = (0, 1, -1)$  are given vectors, then a vector  $\vec{B}$  satisfying the equations  $\vec{A} \times \vec{B} = \vec{C}$  and  $\vec{A} \cdot \vec{B} = 3$  \_\_\_\_\_. [1985 - 2 Marks]

39. If  $\vec{A}, \vec{B}, \vec{C}$  are three non-coplanar vectors, then  $-\frac{\vec{A} \cdot \vec{B} \times \vec{C}}{\vec{C} \cdot \vec{A} \times \vec{B}} + \frac{\vec{B} \cdot \vec{A} \times \vec{C}}{\vec{C} \cdot \vec{A} \times \vec{B}} = \dots$ . [1985 - 2 Marks]

40. If  $\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0$  and the vectors  $\vec{A} = (1, a, a^2), \vec{B} = (1, b, b^2), \vec{C} = (1, c, c^2)$ , are non-coplanar, then the product  $abc = \dots$ . [1985 - 2 Marks]

41. The area of the triangle whose vertices are  $A(1, -1, 2), B(2, 1, -1), C(3, -1, 2)$  is \_\_\_\_\_. [1983 - 1 Mark]

**5 True / False**

42. For any three vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$ ,  $(\vec{a} - \vec{b}) \cdot (\vec{b} - \vec{c}) \times (\vec{c} - \vec{a}) = 2\vec{a} \cdot \vec{b} \times \vec{c}$ . [1989 - 1 Mark]

43. If  $X \cdot A = 0, X \cdot B = 0, X \cdot C = 0$  for some non-zero vector  $X$ , then  $[A B C] = 0$ . [1983 - 1 Mark]

44. Let  $\vec{A}, \vec{B}$  and  $\vec{C}$  be unit vectors suppose that  $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C} = 0$ , and that the angle between  $\vec{B}$  and  $\vec{C}$  is  $\pi/6$ . Then  $\vec{A} = \pm 2(\vec{B} \times \vec{C})$ . [1981 - 2 Marks]

**6 MCQs with One or More than One Correct Answer**

45. Let  $\hat{i}, \hat{j}$  and  $\hat{k}$  be the unit vectors along the three positive coordinate axes. Let  $\vec{a} = 3\hat{i} + \hat{j} - \hat{k}, \vec{b} = \hat{i} + b_2\hat{j} + b_3\hat{k}, \vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}, b_2, b_3 \in \mathbb{R}, c_1, c_2, c_3 \in \mathbb{R}$  be three vectors such that  $b_2b_3 > 0, \vec{a} \cdot \vec{b} = 0$  and

$$\begin{pmatrix} 0 & -c_3 & c_2 \\ c_3 & 0 & -c_1 \\ -c_2 & c_1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 3-c_1 \\ 1-c_2 \\ -1-c_3 \end{pmatrix}$$

Then, which of the following is/are TRUE ?

- (a)  $\vec{a} \cdot \vec{c} = 0$  (b)  $\vec{b} \cdot \vec{c} = 0$   
 (c)  $|\vec{b}| > \sqrt{10}$  (d)  $|\vec{c}| \leq \sqrt{11}$

46. Let  $O$  be the origin and

$$\vec{OA} = 2\hat{i} + 2\hat{j} + \hat{k}, \vec{OB} = \hat{i} - 2\hat{j} + 2\hat{k} \text{ and}$$

$$\vec{OC} = \frac{1}{2}(\vec{OB} - \lambda\vec{OA}) \text{ for some } \lambda > 0. \text{ If}$$

$$|\vec{OB} \times \vec{OA}| = \frac{9}{2}, \text{ then which of the following}$$

statements is (are) TRUE? [Adv. 2021]

- (a) Projection of  $\vec{OC}$  on  $\vec{OA}$  is  $-\frac{3}{2}$   
 (b) Area of the triangle  $OAB$  is  $\frac{9}{2}$   
 (c) Area of the triangle  $ABC$  is  $\frac{9}{2}$   
 (d) The acute angle between the diagonals of the parallelogram with adjacent sides  $\vec{OA}$  and  $\vec{OC}$  is  $\frac{\pi}{3}$

47. Let  $a$  and  $b$  be positive real numbers. Suppose

$\vec{PQ} = a\hat{i} + b\hat{j}$  and  $\vec{PS} = a\hat{i} - b\hat{j}$  are adjacent sides of a parallelogram  $PQRS$ . Let  $\vec{u}$  and  $\vec{v}$  be the projection vectors of  $\vec{w} = \hat{i} + \hat{j}$  along  $\vec{PQ}$  and  $\vec{PS}$ , respectively.

If  $|\vec{u}| + |\vec{v}| = |\vec{w}|$  and if the area of the parallelogram  $PQRS$  is 8, then which of the following statements is/are TRUE?

- (a)  $a + b = 4$  [Adv. 2020]  
 (b)  $a - b = 2$   
 (c) The length of the diagonal  $PR$  of the parallelogram  $PQRS$  is 4  
 (d)  $\vec{w}$  is an angle bisector of the vectors  $\vec{PQ}$  and  $\vec{PS}$

48. Let  $\Delta PQR$  be a triangle. Let  $\vec{a} = \vec{QR}$ ,  $\vec{b} = \vec{RP}$  and  $\vec{c} = \vec{PQ}$ .

If  $|\vec{a}| = 12$ ,  $|\vec{b}| = 4\sqrt{3}$ ,  $\vec{b} \cdot \vec{c} = 24$ , then which of the following is (are) true? [JEE Adv. 2015]

- (a)  $\frac{|\vec{c}|^2}{2} - |\vec{a}| = 12$  (b)  $\frac{|\vec{c}|^2}{2} + |\vec{a}| = 30$   
 (c)  $|\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 48\sqrt{3}$  (d)  $\vec{a} \cdot \vec{b} = -72$

49. Let  $\vec{x}$ ,  $\vec{y}$  and  $\vec{z}$  be three vectors each of magnitude  $\sqrt{2}$  and the angle between each pair of them is  $\frac{\pi}{3}$ . If  $\vec{a}$  is

a non-zero vector perpendicular to  $\vec{x}$  and  $\vec{y} \times \vec{z}$  and  $\vec{b}$  is a non-zero vector perpendicular to  $\vec{y}$  and  $\vec{z} \times \vec{x}$ , then [Adv. 2014]

(a)  $\vec{b} = \begin{pmatrix} \vec{x} \cdot \vec{z} \\ \vec{b} \cdot \vec{z} \end{pmatrix} \begin{pmatrix} \vec{x} - \vec{y} \end{pmatrix}$

(b)  $\vec{a} = \begin{pmatrix} \vec{x} \cdot \vec{z} \\ \vec{a} \cdot \vec{y} \end{pmatrix} \begin{pmatrix} \vec{x} - \vec{z} \end{pmatrix}$

(c)  $\vec{a} \cdot \vec{b} = - \begin{pmatrix} \vec{x} \cdot \vec{z} \\ \vec{a} \cdot \vec{y} \end{pmatrix} \begin{pmatrix} \vec{x} - \vec{z} \end{pmatrix}$

(d)  $\vec{a} = - \begin{pmatrix} \vec{x} \cdot \vec{z} \\ \vec{a} \cdot \vec{y} \end{pmatrix} \begin{pmatrix} \vec{x} - \vec{z} \end{pmatrix}$

50. The vector (s) which is/are coplanar with vectors  $\hat{i} + \hat{j} + 2\hat{k}$  and  $\hat{i} + 2\hat{j} + \hat{k}$ , and perpendicular to the vector  $\hat{i} + \hat{j} + \hat{k}$  is/are [2011]

- (a)  $\hat{j} - \hat{k}$  (b)  $-\hat{i} + \hat{j}$   
 (c)  $\hat{i} - \hat{j}$  (d)  $-\hat{j} + \hat{k}$

51. Let  $a$  and  $b$  be two non-collinear unit vectors. If  $u = a - (a \cdot b)b$  and  $v = a \times b$ , then  $|v|$  is [1999 - 3 Marks]

- (a)  $|u|$  (b)  $|u| + |u \cdot a|$   
 (c)  $|u| + |u \cdot b|$  (d)  $|u| + u \cdot (a + b)$

52. Which of the following expressions are meaningful? [1998 - 2 Marks]

- (a)  $u(v \times w)$  (b)  $(u \cdot v) \cdot w$   
 (c)  $(u \cdot v)w$  (d)  $u \times (v \cdot w)$

53. For three vectors  $u, v, w$  which of the following expression is not equal to any of the remaining three? [1998 - 2 Marks]

- (a)  $u \cdot (v \times w)$  (b)  $(v \times w) \cdot u$   
 (c)  $v \cdot (u \times w)$  (d)  $(u \times v) \cdot w$

54. The number of vectors of unit length perpendicular to vectors  $\vec{a} = (1, 1, 0)$  and  $\vec{b} = (0, 1, 1)$  is [1987 - 2 Marks]

- (a) one (b) two  
 (c) three (d) infinite

55. Let  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ,  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$  and  $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$  be three non-zero vectors such that  $\vec{c}$  is a unit vector perpendicular to both the vectors  $\vec{a}$

and  $\vec{b}$ . If the angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{6}$ , then

- $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2$  is equal to [1986 - 2 Marks]  
 (a) 0  
 (b) 1  
 (c)  $\frac{1}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)$   
 (d)  $\frac{3}{4}(a_1^2 + a_2^2 + a_3^2)(b_1^2 + b_2^2 + b_3^2)(c_1^2 + c_2^2 + c_3^2)$



7 Match the Following

56. Match List I with List II and select the correct answer using the code given below the lists : [Adv. 2014]

P. Let  $y(x) = \cos\left(3\cos^{-1}x\right)$ ,  $x \in [-1, 1]$ ,  $x \neq \pm \frac{\sqrt{3}}{2}$ . Then

$$\frac{1}{y(x)} \left\{ (x^2 - 1) \frac{d^2y(x)}{dx^2} + x \frac{dy(x)}{dx} \right\} \text{ equals}$$

Q. Let  $A_1, A_2, \dots, A_n$  ( $n > 2$ ) be the vertices of a regular

polygon of  $n$  sides with its centre at the origin. Let  $\vec{a}_k$  be the position vector of the point  $A_k$ ,  $k = 1, 2, \dots, n$ .

$$\text{If } \left| \sum_{k=1}^{n-1} \begin{pmatrix} \vec{a}_k \times \vec{a}_{k+1} \\ \vec{a}_k \cdot \vec{a}_{k+1} \end{pmatrix} \right| = \left| \sum_{k=1}^{n-1} \begin{pmatrix} \vec{a}_k \times \vec{a}_{k+1} \\ \vec{a}_k \cdot \vec{a}_{k+1} \end{pmatrix} \right|,$$

then the minimum value of  $n$  is

R. If the normal from the point  $P(h, 1)$  on the ellipse

$$\frac{x^2}{6} + \frac{y^2}{3} = 1 \text{ is perpendicular to the line } x + y = 8, \text{ then the value of } h \text{ is}$$

S. Number of positive solutions satisfying the

$$\text{equation } \tan^{-1}\left(\frac{1}{2x+1}\right) + \tan^{-1}\left(\frac{1}{4x+1}\right) = \tan^{-1}\left(\frac{2}{x^2}\right) \text{ is}$$

	P	Q	R	S
(a)	4	3	2	1
(b)	2	4	3	1
(c)	4	3	1	2
(d)	2	4	1	3

57. Match List I with List II and select the correct answer using the code given below the lists : [Adv. 2013]

P. Volume of parallelepiped determined by vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  is 2. Then the volume of the parallelepiped determined by vectors

$$2(\vec{a} \times \vec{b}), 3(\vec{b} \times \vec{c}) \text{ and } 2(\vec{c} \times \vec{a}) \text{ is}$$

Q. Volume of parallelepiped determined by vectors  $\vec{a}, \vec{b}$  and  $\vec{c}$  is 5. Then the volume of the parallelepiped determined by vectors

$$3(\vec{a} + \vec{b}), 3(\vec{b} + \vec{c}) \text{ and } 2(\vec{c} + \vec{a}) \text{ is}$$

R. Area of a triangle with adjacent sides determined by vectors  $\vec{a}$  and  $\vec{b}$  is 20. Then the area of the triangle with adjacent sides determined

$$\text{by vectors } (2\vec{a} + 3\vec{b}) \text{ and } (\vec{a} - \vec{b}) \text{ is}$$

S. Area of a parallelogram with adjacent sides determined by vectors  $\vec{a}$  and  $\vec{b}$  is 30. Then the area of the parallelogram with adjacent

$$\text{sides determined by vectors } (\vec{a} + \vec{b}) \text{ and } \vec{a} \text{ is}$$

Codes:

	P	Q	R	S
(a)	4	2	3	1
(b)	2	3	1	4
(c)	3	4	1	2
(d)	1	4	3	2

58. Match the statements / expressions given in Column-I with the values given in Column-II. [2009]

(A) Root(s) of the equation  $2 \sin^2 \theta + \sin^2 2\theta = 2$

(p)  $\frac{\pi}{6}$

(B) Points of discontinuity of the function  $f(x) = \left[ \frac{6x}{\pi} \right] \cos \left[ \frac{3x}{\pi} \right]$ , where  $[y]$  denotes the largest integer less than or equal to  $y$

(q)  $\frac{\pi}{4}$

- (C) Volume of the parallelepiped with its edges represented by the vectors  $\hat{i} + \hat{j}, \hat{i} + 2\hat{j}$  and  $\hat{i} + \hat{j} + \pi\hat{k}$  (r)  $\frac{\pi}{3}$
- (D) Angle between vector  $\vec{a}$  and  $\vec{b}$  where  $\vec{a}, \vec{b}$  and  $\vec{c}$  are unit vectors satisfying  $\vec{a} + \vec{b} + \sqrt{3}\vec{c} = \vec{0}$  (s)  $\frac{\pi}{2}$

**8 Comprehension/Passage Based Questions**

Let O be the origin, and  $\vec{OX}, \vec{OY}, \vec{OZ}$  be three unit vectors in the directions of the sides  $\vec{QR}, \vec{RP}, \vec{PQ}$  respectively, of a triangle PQR. [Adv. 2017]

59.  $|\vec{OX} \times \vec{OY}| =$
- (a)  $\sin(P+Q)$  (b)  $\sin 2R$   
 (c)  $\sin(P+R)$  (d)  $\sin(Q+R)$
60. If the triangle PQR varies, then the minimum value of  $\cos(P+Q) + \cos(Q+R) + \cos(R+P)$  is
- (a)  $-\frac{5}{3}$  (b)  $-\frac{3}{2}$  (c)  $\frac{3}{2}$  (d)  $\frac{5}{3}$

**9 Assertion and Reason/Statements Type Questions**

61. Let the vectors  $\vec{PQ}, \vec{QR}, \vec{RS}, \vec{ST}, \vec{TU}$  and  $\vec{UP}$  represent the sides of a regular hexagon.

**STATEMENT-1:**  $\vec{PQ} \times (\vec{RS} + \vec{ST}) \neq \vec{0}$ . because

**STATEMENT-2:**  $\vec{PQ} \times \vec{RS} = \vec{0}$  and  $\vec{PQ} \times \vec{ST} \neq \vec{0}$ .

[2007 -3 marks]

- (a) Statement-1 is True, statement-2 is True; Statement-2 is a correct explanation for Statement-1.  
 (b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1  
 (c) Statement-1 is True, Statement-2 is False  
 (d) Statement-1 is False, Statement-2 is True.

**10 Subjective Problems**

62. If  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  are distinct vectors such that  $\vec{a} \times \vec{c} = \vec{b} \times \vec{d}$  and  $\vec{a} \times \vec{b} = \vec{c} \times \vec{d}$ . Prove that  $(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) \neq 0$  i.e.  $\vec{a}\vec{b} + \vec{d}\vec{c} \neq \vec{d}\vec{b} + \vec{a}\vec{c}$  [2004 - 2 Marks]
63. If  $\vec{u}, \vec{v}, \vec{w}$ , are three non-coplanar unit vectors and  $\alpha, \beta, \gamma$  are the angles between  $\vec{u}$  and  $\vec{v}$  and  $\vec{w}$ ,  $\vec{v}$  and  $\vec{u}$  respectively and  $\vec{x}, \vec{y}, \vec{z}$  are unit vectors along the bisectors of the angles  $\alpha, \beta, \gamma$  respectively. Prove that  $[\vec{x} \times \vec{y} \cdot \vec{y} \times \vec{z} \cdot \vec{z} \times \vec{x}] = \frac{1}{16} [\vec{u} \cdot \vec{v} \cdot \vec{w}]^2 \sec^2 \frac{\alpha}{2} \sec^2 \frac{\beta}{2} \sec^2 \frac{\gamma}{2}$ . [2003 - 4 Marks]

64. Let  $V$  be the volume of the parallelepiped formed by the vectors  $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ ,  $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ ,  $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ . If  $a_r, b_r, c_r$ , where  $r=1, 2, 3$ , are non-negative real numbers and  $\sum_{r=1}^3 (a_r + b_r + c_r) = 3L$ , show

that  $V \leq L^3$ . [2002 - 5 Marks]

65. Let  $u$  and  $v$  be unit vectors. If  $w$  is a vector such that  $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$ , then prove that  $|(\vec{u} \times \vec{v}) \cdot \vec{w}| \leq 1/2$  and that the equality holds if and only if  $u$  is perpendicular to  $v$ . [1999 - 10 Marks]

66. For any two vectors  $u$  and  $v$ , prove that [1998 - 8 Marks]

- (a)  $(\vec{u} \cdot \vec{v})^2 + |\vec{u} \times \vec{v}|^2 = |\vec{u}|^2 |\vec{v}|^2$  and  
 (b)  $(1 + |\vec{u}|^2)(1 + |\vec{v}|^2) = (1 - \vec{u} \cdot \vec{v})^2 + |\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2$ .

67. If  $A, B$  and  $C$  are vectors such that  $|\vec{B}| = |\vec{C}|$ . Prove that  $[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} \times \vec{C}) \cdot (\vec{B} + \vec{C}) = 0$ .

[1997 - 5 Marks]

68. The position vectors of the vertices  $A, B$  and  $C$  of a tetrahedron  $ABCD$  are  $\hat{i} + \hat{j} + \hat{k}, \hat{i}$  and  $3\hat{i}$ , respectively. The altitude from vertex  $D$  to the opposite face  $ABC$  meets the median line through  $A$  of the triangle  $ABC$  at a point  $E$ . If the length of the side  $AD$  is 4 and the volume of the

tetrahedron is  $\frac{2\sqrt{2}}{3}$ , find the position vector of the point  $E$  for all its possible positions. [1996 - 5 Marks]

69. If the vectors  $\vec{b}, \vec{c}, \vec{d}$ , are not coplanar, then prove that the vector

$(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) + (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) + (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c})$  is parallel to  $\vec{a}$ . [1994 - 4 Marks]

70. Let  $\vec{A} = 2\vec{i} + \vec{k}, \vec{B} = \vec{i} + \vec{j} + \vec{k}$ , and  $\vec{C} = 4\vec{i} - 3\vec{j} + 7\vec{k}$ . Determine a vector  $\vec{R}$ . Satisfying  $\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$  and  $\vec{R} \cdot \vec{A} = 0$  [1990 - 3 Marks]

71. If vectors  $\vec{a}, \vec{b}, \vec{c}$  are coplanar, show that

$$\begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \end{vmatrix} = \vec{0} \quad [1989 - 2 Marks]$$

72. If  $A, B, C, D$  are any four points in space, prove that –  
[1987 - 2 Marks]

$$\left| \overline{AB} \times \overline{CD} + \overline{BC} \times \overline{AD} + \overline{CA} \times \overline{BD} \right| = 4 \text{ (area of triangle } ABC)$$

73. The position vectors of the points  $A, B, C$  and  $D$  are  $3\hat{i} - 2\hat{j} - \hat{k}$ ,  $2\hat{i} + 3\hat{j} - 4\hat{k}$ ,  $-\hat{i} + \hat{j} + 2\hat{k}$  and  $4\hat{i} + 5\hat{j} + \lambda\hat{k}$ , respectively. If the points  $A, B, C$  and  $D$  lie on a plane, find the value of  $\lambda$ . [1986 - 2½ Marks]

74. Find all values of  $\lambda$  such that  $x, y, z, \neq (0, 0, 0)$  and  $(\hat{i} + \hat{j} + 3\hat{k})x + (3\hat{i} - 3\hat{j} + \hat{k})y + (-4\hat{i} + 5\hat{j})z = \lambda(x\hat{i} + \hat{j}y + \hat{k}z)$  where  $\hat{i}, \hat{j}, \hat{k}$  are unit vectors along the coordinate axes. [1982 - 3 Marks]

75.  $A_1, A_2, \dots, A_n$  are the vertices of a regular plane polygon with  $n$  sides and  $O$  is its centre. Show that  $\sum_{i=1}^{n-1} (\overline{OA}_i \times \overline{OA}_{i+1}) = (1-n)(\overline{OA}_2 \times \overline{OA}_1)$  [1982 - 2 Marks]



Answer Key

**Topic-1 : Algebra of Vectors, Linear Dependence & Independence of Vectors, Vector Inequality**

1. (a)    2. (b)    3. (a)    4. (5)    5. True    6. (d)    7. (A)-p, r, s; (B)-p; (C)-p, q; (D)-s, t  
8. (A)-q; (B)-p, q; (C)-p, q, s, t; (D)-q, t

**Topic-2 : Scalar or Dot Product of two Vectors**

1. (c)    2. (a)    3. (b)    4. (b)    5. (b)    6. (b)    7. (a)    8. (2)    9. (3)  
10.  $\left( \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \vec{b}, \vec{a} - \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \vec{b}$     11.  $(2i - j)$     12. orthocentre    13.  $5\sqrt{2}$     14. (a, c, d)  
15. (a, c)    16. A-q, B-p, C-s, D-t

**Topic-3 : Vector or Cross Product of two vectors, Scalar & Vector Triple Product**

1. (b)    2. (c)    3. (b)    4. (c)    5. (a)    6. (b)    7. (c)    8. (a)    9. (c)    10. (c)  
11. (c)  
12. (c)    13. (c)    14. (a)    15. (a)    16. (b)    17. (a)    18. (b)    19. (d)    20. (a)    21. (a)  
22. (b)    23. (d)    24. (d)    25. (d)    26. (a)    27. (7)    28. (3)    29. (9)    30. (4)    31. (9)  
32. (5)    33. (18)    34. (6)    35.  $(\hat{a})$     36.  $\frac{\hat{j} - \hat{k}}{\sqrt{2}}, \frac{-\hat{j} + \hat{k}}{\sqrt{2}}$     37. (1)    38.  $\frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$   
39. (0)    40. (-1)    41.  $\sqrt{13}$     42. False    43. True    44. True    45. (b,c,d)    46. (a,b,c)    47. (b, c, d)  
48. (a,c,d)    49. (a,b,c)    50. (a,d)    51. (a, c)    52. (a, c)    53. (c)    54. (b)    55. (c)    56. (a)    57. (c)  
58. (A)-q,s; (B)-p,r,s,t; (C)-t; (D)-r    59. (a)    60. (b)    61. (c)



# Hints & Solutions

## Topic-1: Algebra of Vectors, Linear Dependence & Independence of Vectors, Vector Inequality

- (a) Since  $\overline{PQ} = 6\hat{i} + \hat{j}$ ,  $\overline{QR} = -\hat{i} + 3\hat{j}$ ,  $\overline{SR} = 6\hat{i} + \hat{j}$ ,  
 $\overline{PS} = -\hat{i} + 3\hat{j}$ .  
 Here  $\overline{PQ} = \overline{SR}$ ;  $\overline{QR} = \overline{PS}$  and  $\overline{PQ} \cdot \overline{PS} \neq 0$   
 Also  $|\overline{PQ}| \neq |\overline{QR}|$   
 $\Rightarrow$  PQRS is a parallelogram but neither a rhombus nor a rectangle.

(b) Let  $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ ,  $\beta\hat{i} + \gamma\hat{j} + \alpha\hat{k}$  and  $\gamma\hat{i} + \alpha\hat{j} + \beta\hat{k}$  be position vectors of points A, B and C respectively, then  
 $|\overline{AB}| = \sqrt{(\beta - \alpha)^2 + (\gamma - \beta)^2 + (\alpha - \gamma)^2}$   
 $|\overline{BC}| = \sqrt{(\gamma - \beta)^2 + (\alpha - \gamma)^2 + (\alpha - \beta)^2}$   
 $|\overline{CA}| = \sqrt{(\alpha - \gamma)^2 + (\beta - \alpha)^2 + (\gamma - \beta)^2}$   
 $\therefore |\overline{AB}| = |\overline{BC}| = |\overline{CA}|$   
 $\Rightarrow \Delta ABC$  is an equilateral triangle.
- (a) Let  $60\hat{i} + 3\hat{j}$ ,  $40\hat{i} - 8\hat{j}$  and  $a\hat{i} - 52\hat{j}$  be position vector of points A, B and C respectively.  
 $\therefore \overline{AB} = 40\hat{i} - 8\hat{j} - 60\hat{i} - 3\hat{j} = -20\hat{i} - 11\hat{j}$   
 and  $\overline{AC} = a\hat{i} - 52\hat{j} - 60\hat{i} - 3\hat{j} = (a - 60)\hat{i} - 55\hat{j}$   
 Given that A, B and are collinear  
 $\therefore \overline{AB} \parallel \overline{AC} \Rightarrow \frac{a - 60}{-20} = \frac{-55}{-11} \Rightarrow a = -40$

(5) Given 8 vectors are  
 $(1, 1, 1)$ ,  $(-1, -1, -1)$ ;  $(-1, 1, 1)$ ,  $(1, -1, -1)$ ;  $(1, -1, 1)$ ,  
 $(-1, 1, -1)$ ;  $(1, 1, -1)$ ,  $(-1, -1, 1)$   
 These are 4 diagonals of a cube and their opposites.  
 For 3 non coplanar vectors first we select 3 groups of diagonals and its opposite in  ${}^4C_3$  ways. Then one vector from each group can be selected in  $2 \times 2 \times 2$  ways.  
 $\therefore$  Total ways =  ${}^4C_3 \times 2 \times 2 \times 2 = 32 = 2^5$   
 $\therefore p = 5$

True: Let  $\vec{a} + \vec{b}$ ,  $\vec{a} - \vec{b}$  and  $\vec{a} + k\vec{b}$  be position vectors of points A, B and C respectively.  
 Then,  $\overline{AB} = (\vec{a} - \vec{b}) - (\vec{a} + \vec{b}) = -2\vec{b}$   
 and  $\overline{BC} = \vec{a} + k\vec{b} - \vec{a} + \vec{b} = (k + 1)\vec{b}$   
 $\Rightarrow \overline{AB} \parallel \overline{BC} \quad \forall k \in R$   
 $\Rightarrow A, B, C$  are collinear  $\forall k \in R$   
 $\therefore$  Statement is true.

- (d) Given that,  $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$   
 and  $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$  are linearly dependent,

$$\therefore [\vec{a} \vec{b} \vec{c}] = 0$$

$$\Rightarrow \begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & 4 \\ 1 & \alpha & \beta \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 - R_1$  and  $R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 1 & 0 & 0 \\ 4 & -1 & 0 \\ 1 & \alpha - 1 & \beta - 1 \end{vmatrix} = 0$$

$$\Rightarrow \beta - 1 = 0 \Rightarrow \beta = 1$$

Also given that  $|\vec{c}| = \sqrt{3} \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

Substituting the value of  $\beta$  we get  $\alpha^2 = 1$

$$\Rightarrow \alpha = \pm 1$$

- (A)  $\rightarrow p, r, s$ ; (B)  $\rightarrow p$ ; (C)  $\rightarrow p, q$ ; (D)  $\rightarrow s, t$

(A) Since,  $2(a^2 - b^2) = c^2$

$$\Rightarrow 2(\sin^2 x - \sin^2 y) = \sin^2 z$$

$$\Rightarrow 2\sin(x + y)\sin(x - y) = \sin^2 z$$

$$\Rightarrow 2\sin(x - y) = \sin z \quad (\because \sin(x + y) = \sin z)$$

$$\Rightarrow \frac{\sin(x - y)}{\sin z} = \frac{1}{2} = \lambda \quad (\text{Given})$$

$$\therefore \cos(n\pi\lambda) = 0 \Rightarrow \cos \frac{n\pi}{2} = 0 \Rightarrow n = 1, 3, 5$$

(B)  $1 + \cos 2X - 2\cos 2Y = 2\sin X \sin Y$

$$\Rightarrow 2\cos^2 X - 2(1 - 2\sin^2 Y) = 2\sin X \sin Y$$

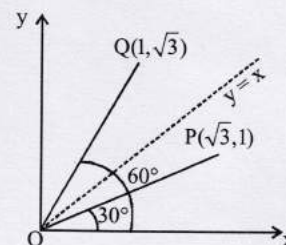
$$\Rightarrow 1 - \sin^2 X - 1 + 2\sin^2 Y = \sin X \sin Y$$

$$\Rightarrow \sin^2 X + \sin X \sin Y - 2\sin^2 Y = 0$$

$$\Rightarrow (\sin X - \sin Y)(\sin X + 2\sin Y) = 0$$

$$\Rightarrow \frac{\sin X}{\sin Y} = 1 \text{ or } -2 \therefore \frac{a}{b} = 1.$$

- (C)  $P(\sqrt{3}, 1)$ ,  $Q(1, \sqrt{3})$ ,  $R(\beta, 1 - \beta)$



From figure, acute angle bisector of  $\angle XOY$  is  $y = x$ .  
 $\therefore$  Distance of  $R(\beta, 1 - \beta)$  from bisector

$$= \left| \frac{\beta - 1 + \beta}{\sqrt{2}} \right| = \frac{3}{\sqrt{2}}$$

$$\Rightarrow 2\beta - 1 = \pm 3 \text{ or } \beta = 2 \text{ or } -1$$

$\therefore |\beta| = 1, 2$

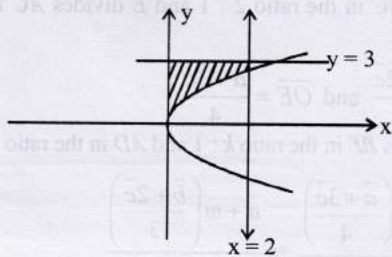
(D) For  $\alpha = 0, y = 3$

For  $\alpha = 1, y = |x - 1| + |x - 2| + x$   
 $\alpha = 0$

**Case I**

$F(\alpha)$  is the area bounded by  $x = 0, x = 2, y^2 = 4x$  and  $y = 3$

$\therefore F(\alpha) = \int_0^2 (3 - 2\sqrt{x}) dx$



$$= \left[ 3x - \frac{4x\sqrt{x}}{3} \right]_0^2 = 6 - \frac{8\sqrt{2}}{3}$$

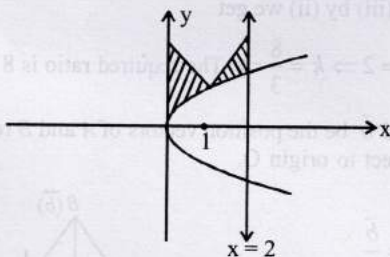
$\therefore F(\alpha) + \frac{8}{3}\sqrt{2} = 6$

**Case II**

$F(\alpha)$  is the area bounded by  $x = 0, x = 2, y^2 = 4x$  and  $y = |x - 1| + |x - 2| + x$

$$= \begin{cases} 3 - x, 0 \leq x < 1 \\ x + 1, 1 \leq x < 2 \\ 3x - 3, x \geq 2 \end{cases}$$

$\therefore F(\alpha) = \int_0^1 (3 - x - 2\sqrt{x}) dx + \int_1^2 (x + 1 - 2\sqrt{x}) dx$



$$= \left( 3x - \frac{x^2}{2} - \frac{4x}{3}\sqrt{x} \right)_0^1 + \left( \frac{x^2}{2} + x - \frac{4}{3}x\sqrt{x} \right)_1^2$$

$$= 3 - \frac{1}{2} - \frac{4}{3} + 2 + 2 - \frac{8\sqrt{2}}{3} - \frac{1}{2} - 1 + \frac{4}{3} = 5 - \frac{8\sqrt{2}}{3}$$

$F(\alpha) + \frac{8\sqrt{2}}{3} = 5$

8. (A)  $\rightarrow$  q; (B)  $\rightarrow$  p, q; (C)  $\rightarrow$  p, q, s, t; (D)  $\rightarrow$  q, t

(A) Projection of  $\alpha\hat{i} + \beta\hat{j}$  on  $\sqrt{3}\hat{i} + \hat{j} = \frac{\sqrt{3}\alpha + \beta}{2} = \sqrt{3}$

$\Rightarrow \alpha = \frac{2\sqrt{3} - \beta}{\sqrt{3}}$

$\therefore \frac{2\sqrt{3} - \beta}{\sqrt{3}} = 2 + \sqrt{3}\beta \Rightarrow \beta = 0 \Rightarrow \alpha = 2$

(B) LHD =  $f'(1) = -6a$  and RHD =  $f'(1) = b$   
 $-6a = b$  ... (i)

Also  $f$  is continuous at  $x = 1$ ,

$\therefore -3a - 2 = b + a^2$

$\Rightarrow a^2 - 3a + 2 = 0$  (from (i))

$\Rightarrow a = 1, 2$

(C)  $(3 - 3\omega + 2\omega^2)^{4n+3} + (2 + 3\omega - 3\omega^2)^{4n+3} + (-3 + 2\omega + 3\omega^2)^{4n+3} = 0$  [ $\because \omega^3 = 1$ ]

$$\Rightarrow (3 - 3\omega + 2\omega^2)^{4n+3} + \left( \frac{2\omega^2 + 3 - 3\omega}{\omega^2} \right)^{4n+3} + \left( \frac{-3\omega + 2\omega^2 + 3}{\omega} \right)^{4n+3} = 0$$

$\Rightarrow (3 - 3\omega + 2\omega^2)^{4n+3} [1 + \omega^{4n+3} + (\omega^2)^{4n+3}] = 0$

$\Rightarrow 4n + 3$  should be an integer other than multiple of 3.

$\therefore n = 1, 2, 4, 5$

(D)  $\therefore$  H.M of  $a$  and  $b$  be 4.

$\therefore \frac{2ab}{a+b} = 4 \Rightarrow ab = 2a + 2b$  ... (i)

Also  $a + q = 10$  or  $a = 10 - q$   
 and  $b + 5 = 2q$  or  $b = 2q - 5$

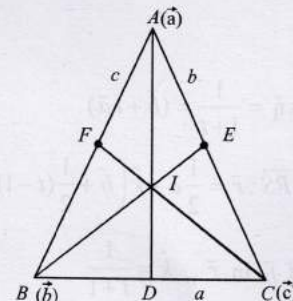
Putting values of  $a$  and  $b$  in eq<sup>n</sup>(i), we get

$q = 4$  or  $\frac{15}{2} \Rightarrow a = 6$  or  $\frac{5}{2}$

$\therefore |q - a| = 2$  or  $5$ .

9. Let  $\vec{a}, \vec{b}, \vec{c}$  be the position vectors of vertices  $A, B$  and  $C$  respectively with respect to origin,

Let  $AD, BE$  and  $CF$  be the bisectors of  $\angle A, \angle B$ , and  $\angle C$  respectively.



Let  $a, b, c$  are the lengths of sides  $BC, CA$  and  $AB$  respectively, we know that by angle bisector theorem  $BD : DC = AB : AC = c : b$ .

$\therefore$  The position vector of  $D$  is  $\vec{d} = \frac{b\vec{b} + c\vec{c}}{b+c}$

Let  $I$  be the point of intersection of  $BE$  and  $AD$ . Then in  $\triangle ABD$ ,  $BI$  is bisector of  $\angle B$ .

$\therefore DI : IA = BD : BA$

But  $\frac{BD}{DC} = \frac{c}{b} \Rightarrow \frac{BD}{BD+DC} = \frac{c}{c+b}$

$\Rightarrow \frac{BD}{BC} = \frac{c}{c+b} \Rightarrow BD = \frac{ac}{b+c}$

$\therefore DI : IA = \frac{ac}{b+c} : c = a : (b+c)$

$\therefore$  P.V. of  $I = \frac{\vec{a} \cdot a + \vec{d}(b+c)}{a+b+c}$

$a\vec{a} + \left(\frac{b\vec{b} + c\vec{c}}{b+c}\right)(b+c) = \frac{a\vec{a} + b\vec{b} + c\vec{c}}{a+b+c}$

Similarly p.v. of intersection of

$AD$  and  $CF$  is also  $\frac{a\vec{a} + b\vec{b} + c\vec{c}}{a+b+c}$

Hence all the  $\angle$  bisectors passes through  $I$ , i.e., these are concurrent.

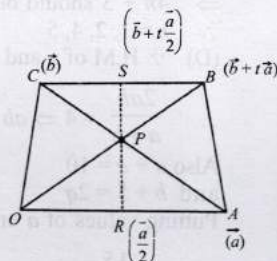
10. Let  $OABC$  is trapezium and position vector of  $A, B, C$  with respect to origin  $O$  are  $A(\vec{a}), C(\vec{b}), B(\vec{b} + t\vec{a})$

Equation of  $\overline{OB}$

$\vec{r} = \lambda(\vec{b} + t\vec{a})$

Equation of  $AC$  :

$\vec{r} = \vec{a} + \mu(\vec{b} - \vec{a})$



Let  $P$  be the point of intersection  $\overline{OB}$  and  $\overline{AC}$

$\therefore \lambda(\vec{b} + t\vec{a}) = \vec{a} + \mu(\vec{b} - \vec{a})$

On comparing both sides

$\lambda = \mu \dots (i)$

$\lambda t = 1 - \mu \dots (ii)$

from (i) and (ii)

$\lambda = \frac{1}{1+t}$

$\therefore$  P.V. of  $P$  is  $\vec{r}_1 = \frac{1}{1+t}(\vec{b} + t\vec{a})$

Equation of  $\overline{RS} : \vec{r} = \frac{1}{2}\vec{a} + \hat{k}\left(\vec{b} + \frac{1}{2}(t-1)\vec{a}\right)$

Coefficient of  $\vec{b}$  in  $\vec{r}_1, \hat{k} = \frac{1}{t+1}$

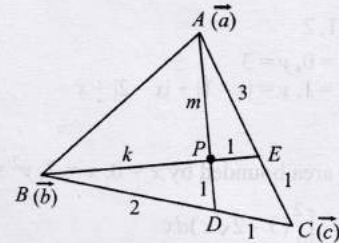
$\therefore \vec{r} = \frac{1}{2}\vec{a} + \frac{1}{t+1}\left[\vec{b} + \frac{1}{2}(t-1)\vec{a}\right]$

$= \frac{1}{t+1}\vec{b} + \frac{1}{2(t+1)}[t-1+t+1]\vec{a} = \frac{1}{t+1}[\vec{b} + t\vec{a}] = \vec{r}_1$

Hence  $p$  lies on  $RS$ .

Hence proved.

11. Let  $\vec{a}, \vec{b}, \vec{c}$ , be the position vectors of points  $A, B$  and  $C$  respectively with respect to origin  $O$ .



$\therefore D$  divides  $BC$  in the ratio  $2 : 1$  and  $E$  divides  $AC$  in the ratio  $3 : 1$ .

$\therefore \overline{OD} = \frac{\vec{b} + 2\vec{c}}{3}$  and  $\overline{OE} = \frac{\vec{a} + 3\vec{c}}{4}$

Let pt.  $P$  divides  $BE$  in the ratio  $k : 1$  and  $AD$  in the ratio  $m : 1$ ,

$\therefore \overline{OP} = \frac{\vec{b} + k\left(\frac{\vec{a} + 3\vec{c}}{4}\right)}{k+1} = \frac{\vec{a} + m\left(\frac{\vec{b} + 2\vec{c}}{3}\right)}{m+1}$

$\Rightarrow \frac{k}{4(k+1)}\vec{a} + \frac{1}{k+1}\vec{b} + \frac{3k}{4(k+1)}\vec{c}$

$= \frac{1}{m+1}\vec{a} + \frac{m}{3(m+1)}\vec{b} + \frac{2m}{3(m+1)}\vec{c}$

On comparing both sides, we get

$\Rightarrow \frac{k}{4(k+1)} = \frac{1}{m+1} \dots (i)$

$\frac{1}{k+1} = \frac{m}{3(m+1)} \dots (ii)$

$\frac{3k}{4(k+1)} = \frac{2m}{3(m+1)} \dots (iii)$

Dividing (iii) by (ii) we get

$\frac{3k}{4} = 2 \Rightarrow k = \frac{8}{3} \Rightarrow$  The required ratio is  $8 : 3$ .

12. Let  $\vec{a}$  and  $\vec{b}$  be the position vectors of  $A$  and  $B$  respectively with respect to origin  $O$ .

$\Rightarrow \overline{OE} = \frac{\vec{b}}{2}$

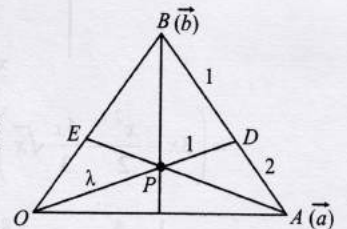
$\overline{OD} = \frac{1\vec{a} + 2\vec{b}}{1+2} = \frac{\vec{a} + 2\vec{b}}{3}$

$\therefore$  Equation of  $OD$  is

$\vec{r} = t\left(\frac{\vec{a} + 2\vec{b}}{3}\right) \dots (i)$

and Equation of  $AE$  is

$\vec{r} = \vec{a} + s\left(\frac{\vec{b}}{2} - \vec{a}\right) \dots (ii)$



Since  $OD$  and  $AE$  intersect at  $P$ , then comparing the coefficients of  $\vec{a}$  and  $\vec{b}$ , we get

$$\frac{t}{3} = 1 - s \quad \text{and} \quad \frac{2t}{3} = \frac{s}{2} \Rightarrow t = \frac{3}{5} \quad \text{and} \quad s = \frac{4}{5}$$

Putting value of  $t$  in eq. (i) we get position vector of point of intersection  $P$  is  $\frac{\vec{a} + 2\vec{b}}{5}$  ... (iii)

Let  $P$  divides  $OD$  in the ratio  $\lambda : 1$ , then position vector of  $P$  is

$$\frac{\lambda \left( \frac{\vec{a} + 2\vec{b}}{3} \right) + 1 \cdot 0}{\lambda + 1} = \frac{\lambda}{3(\lambda + 1)} (\vec{a} + 2\vec{b}) \quad \dots \text{(iv)}$$

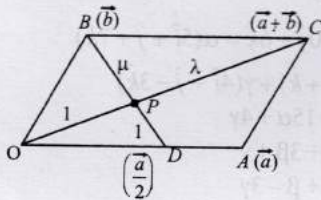
From (iii) and (iv) we get

$$\frac{\lambda}{3(\lambda + 1)} = \frac{1}{5} \Rightarrow 5\lambda = 3\lambda + 3 \Rightarrow \lambda = 3/2$$

$$\therefore OP : PD = 3 : 2$$

13. Given  $OACB$  is a parallelogram with  $O$  as origin.

$$\text{Let } \vec{OA} = \vec{a}, \vec{OB} = \vec{b} \Rightarrow \vec{OC} = \vec{a} + \vec{b} \text{ and } \vec{OD} = \frac{\vec{a}}{2}$$



Let  $\vec{CO}$  and  $\vec{BD}$  intersect each other at  $P$ .

Let  $P$  dividing  $CO$  in ratio  $\lambda : 1$

$$\vec{OP} = \frac{\lambda \times 0 + 1 \times (\vec{a} + \vec{b})}{\lambda + 1} = \frac{(\vec{a} + \vec{b})}{\lambda + 1} \quad \dots \text{(i)}$$

And  $P$  dividing  $BD$  in the ratio  $\mu : 1$

$$\vec{OP} = \frac{\mu \left( \frac{\vec{a}}{2} \right) + 1(\vec{b})}{\mu + 1} = \frac{\mu\vec{a} + 2\vec{b}}{2(\mu + 1)} \quad \dots \text{(ii)}$$

From (i) and (ii)

$$\frac{\vec{a} + \vec{b}}{\lambda + 1} = \frac{\mu\vec{a} + 2\vec{b}}{2(\mu + 1)}$$

Equating the coefficients of  $\vec{a}$  and  $\vec{b}$ , we get

$$\frac{1}{\lambda + 1} = \frac{\mu}{2(\mu + 1)} \quad \dots \text{(iii)}$$

$$\frac{1}{\lambda + 1} = \frac{1}{\mu + 1} \quad \dots \text{(iv)}$$

From (iv) we get  $\lambda = \mu \Rightarrow P$  divides  $CO$  and  $BD$  in the same ratio.

Putting  $\lambda = \mu$  in eq. (iii) we get  $\mu = 2$

Thus required ratio is  $2 : 1$ .

14. Since vector  $\vec{A}$  has components  $A_1, A_2, A_3$ , in the coordinate system  $OXYZ$ ,

$$\therefore \vec{A} = \hat{i}A_1 + \hat{j}A_2 + \hat{k}A_3$$

When given system is rotated through  $\frac{\pi}{2}$ , the new  $x$ -axis is along old  $y$ -axis and new  $y$ -axis is along the old negative  $x$ -axis and  $z$  remains same as before. Hence the components of  $A$  in the new system are  $A_2, -A_1, A_3$ .

$$\therefore \vec{A} \text{ becomes } A_2\hat{i} - A_1\hat{j} + A_3\hat{k}.$$



### Topic-2: Scalar or Dot Product of two Vectors

1. (c) We know that vector in the plane of  $\vec{a}$  and  $\vec{b}$  is

$$\vec{v} = \vec{a} + \lambda\vec{b}$$

$$\Rightarrow \vec{v} = (1 + \lambda)\hat{i} + (1 - \lambda)\hat{j} + (1 + \lambda)\hat{k}$$

$$\therefore \text{Projection of } \vec{v} \text{ on } \vec{c} \text{ is } \frac{1}{\sqrt{3}}$$

$$\therefore \frac{\vec{v} \cdot \vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}} \Rightarrow \frac{(1 + \lambda) - (1 - \lambda) - (1 + \lambda)}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow 1 - \lambda = -1$$

$$\Rightarrow \lambda = 2$$

$$\vec{v} = 3\hat{i} - \hat{j} + 3\hat{k}$$

2. (a) Given,  $\vec{OP} = \hat{a} \cos t + \hat{b} \sin t$

$$|\vec{OP}|^2 = |\hat{a}|^2 \cos^2 t + |\hat{b}|^2 \sin^2 t + 2\hat{a} \cdot \hat{b} \sin t \cos t$$

$$\Rightarrow |\vec{OP}|^2 = \cos^2 t + \sin^2 t + 2\hat{a} \cdot \hat{b} \sin t \cos t$$

$$\Rightarrow |\vec{OP}|^2 = 1 + \hat{a} \cdot \hat{b} \sin 2t$$

$$|\vec{OP}| = \sqrt{1 + \hat{a} \cdot \hat{b} \sin 2t}$$

$$\therefore |\vec{OP}|_{\max} = \sqrt{1 + \hat{a} \cdot \hat{b}} = M \text{ at } \sin 2t = 1 \Rightarrow t = \frac{\pi}{4}$$

$$\therefore (\vec{OP})_{\max} = \frac{\hat{a} + \hat{b}}{\sqrt{2}} \quad \therefore (\widehat{OP})_{\max} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|}$$

$$\text{Hence } \hat{u} = \frac{\hat{a} + \hat{b}}{|\hat{a} + \hat{b}|} \text{ and } M = \sqrt{1 + \hat{a} \cdot \hat{b}}$$

3. (b) We observe that

$$\vec{a}\vec{b}_1 = \vec{a}\vec{b} - \left( \frac{\vec{b}\vec{a}}{|\vec{a}|^2} \right) |\vec{a}|^2 = \vec{a}\vec{b} - \vec{a}\vec{a} = 0 \quad \dots \text{(i)}$$

$$\vec{a}\vec{c}_2 = \vec{a} \cdot \left( \vec{c} - \frac{\vec{c}\vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{c}\vec{b}_1}{|\vec{b}_1|^2} \vec{b}_1 \right)$$

$$= \vec{a}\vec{c} - \frac{\vec{a}\vec{c}}{|\vec{a}|^2} |\vec{a}|^2 - \frac{\vec{c}\vec{b}_1}{|\vec{b}_1|^2} (\vec{a}\vec{b}_1)$$

$$= \vec{a}\vec{c} - \vec{a}\vec{c} - 0 = 0 \quad [\text{from (i)}]$$

$$\text{And } \vec{b}_1 \cdot \vec{c}_2 = \vec{b}_1 \cdot \left( \vec{c} - \frac{\vec{c}\vec{a}}{|\vec{a}|^2} \vec{a} - \frac{\vec{c}\vec{b}_1}{|\vec{b}_1|^2} \vec{b}_1 \right)$$

$$= \vec{b}_1 \cdot \vec{c} - \frac{(\vec{c}\vec{a})(\vec{b}_1\vec{a})}{|\vec{a}|^2} - \frac{\vec{c}\vec{b}_1}{|\vec{b}_1|^2} |\vec{b}_1|^2$$

$$= \vec{b}_1 \cdot \vec{c} - 0 - \vec{b}_1 \cdot \vec{c} = 0 \quad (\text{from (i)})$$

$$\text{Hence } \vec{a}\vec{b}_1 = \vec{a}\vec{c}_2 = \vec{b}_1\vec{c}_2 = 0$$

$\Rightarrow (\vec{a}, \vec{b}_1, \vec{c}_2)$  is a set of orthogonal vectors.

4. (b) Given that  $\vec{a}$  and  $\vec{b}$  are two unit vectors

$$\therefore |\vec{a}| = |\vec{b}| = 1$$

Also, given that  $(\vec{a} + 2\vec{b})$  is perpendicular to  $(5\vec{a} - 4\vec{b})$

$$\therefore (\vec{a} + 2\vec{b}) \cdot (5\vec{a} - 4\vec{b}) = 0$$

$$\Rightarrow 5|\vec{a}|^2 - 8|\vec{b}|^2 - 4\vec{a}\vec{b} + 10\vec{b}\vec{a} = 0$$

$$\Rightarrow 5 - 8 + 6\vec{a}\vec{b} = 0 \Rightarrow 6|\vec{a}||\vec{b}|\cos\theta = 3$$

[where  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ ]

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ$$

5. (b) Since  $\vec{a}, \vec{b}$  and  $\vec{c}$  are units vectors.

$$\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$$

$$\text{Let } x = |\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2$$

$$= |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\vec{b} + |\vec{b}|^2 + |\vec{c}|^2 - 2\vec{b}\vec{c} + |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c}\vec{a}$$

$$= 6 - 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \quad \dots(i)$$

We have

$$|\vec{a} + \vec{b} + \vec{c}| \geq 0 \Rightarrow |\vec{a} + \vec{b} + \vec{c}|^2 \geq 0$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \geq 0$$

$$\Rightarrow 3 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \geq 0$$

$$\Rightarrow 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \geq -3$$

$$\Rightarrow -2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \leq 3$$

$$\Rightarrow 6 - 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) \leq 9$$

$x \leq 9$  (From (i))  $\therefore x$  does not exceed 9

6. (b) Since  $\vec{u} + \vec{v} + \vec{w} = 0$

$$\Rightarrow |\vec{u} + \vec{v} + \vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 + |\vec{w}|^2 + 2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u})$$

$$\Rightarrow 0 = 9 + 16 + 25 + 2(\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u})$$

$$\Rightarrow (\vec{u}\vec{v} + \vec{v}\vec{w} + \vec{w}\vec{u}) = -25$$

7. (a) Since R divide PQ internally in ratio 2 : 3

$$\therefore \vec{OR} = \frac{3\vec{p} + 2\vec{q}}{3 + 2} = \frac{1}{5}(3\vec{p} + 2\vec{q})$$

S divide PQ externally in ratio 2 : 3

$$\therefore \vec{OS} = \frac{3\vec{p} - 2\vec{q}}{3 - 2} = 3\vec{p} - 2\vec{q}$$

Given that  $\vec{OR} \perp \vec{OS} \Rightarrow \vec{OR} \cdot \vec{OS} = 0$

$$\Rightarrow \frac{1}{5}[3\vec{p} + 2\vec{q}] \cdot (3\vec{p} - 2\vec{q}) = 0$$

$$\Rightarrow 9|\vec{p}|^2 = 4|\vec{q}|^2 \Rightarrow 9p^2 = 4q^2$$

8. (2)  $2\vec{p} + \vec{q} = 5\hat{i} + \hat{j} + 7\hat{k}$

$$\vec{p} - 2\vec{q} = 0\hat{i} + 3\hat{j} + \hat{k}$$

$$\vec{p} \times \vec{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \hat{i}(4) - \hat{j}(-1) + \hat{k}(-3)$$

$$= 4\hat{i} + \hat{j} - 3\hat{k}$$

$$15\hat{i} + 10\hat{j} + 6\hat{k} = \alpha(5\hat{i} + \hat{j} + 7\hat{k})$$

$$+ \beta(3\hat{j} + \hat{k}) + \gamma(4\hat{i} + \hat{j} - 3\hat{k})$$

$$\therefore 15 = 15\alpha + 4\gamma \quad \dots(i)$$

$$10 = \alpha + 3\beta + \gamma \quad \dots(ii)$$

$$6 = 7\alpha + \beta - 3\gamma \quad \dots(iii)$$

On solving (i), (ii) and (iii), we get

$$\therefore \alpha = \frac{7}{5}, \beta = \frac{11}{5}, \gamma = 2 \quad \therefore \gamma = 2$$

9. (3)  $\therefore |\vec{a}| = |\vec{b}| = |\vec{c}| = 1$

$$|\vec{a} - \vec{b}|^2 + |\vec{b} - \vec{c}|^2 + |\vec{c} - \vec{a}|^2 = 9$$

$$\Rightarrow 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2) - 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a}) = 9$$

$$\Rightarrow \vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a} = \frac{-3}{2}$$

Also,

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(\vec{a}\vec{b} + \vec{b}\vec{c} + \vec{c}\vec{a})$$

$$= 1 + 1 + 1 + 2 \times \left( -\frac{3}{2} \right) = 0$$

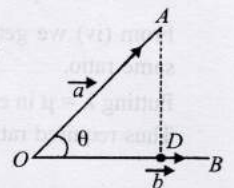
$$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 0 \Rightarrow (\vec{b} + \vec{c}) = -\vec{a}$$

$$\therefore |2\vec{a} + 5(\vec{b} + \vec{c})| = |2\vec{a} - 5\vec{a}| = |-3\vec{a}| = 3$$

10. Component of  $\vec{a}$  along

$$\vec{b} = \vec{OD} = (\text{projection of } \vec{a} \text{ on } \vec{b}) \cdot \hat{b}$$

$$= \left( \frac{\vec{a}\vec{b}}{|\vec{b}|} \right) \frac{\vec{b}}{|\vec{b}|} = \left( \frac{\vec{a}\vec{b}}{|\vec{b}|^2} \right) \vec{b}$$





Component of  $\vec{a}$  perpendicular to  $\vec{b}$

$$= \overline{DA} = \vec{a} - \overline{OD} = \vec{a} - \left( \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2} \right) \vec{b}$$

11. Let  $\vec{c} = a\hat{i} + b\hat{j}$

$$\because \hat{b} \perp \hat{c} \Rightarrow \vec{b} \cdot \vec{c} = 0$$

$$\Rightarrow (4\hat{i} + 3\hat{j}) \cdot (a\hat{i} + b\hat{j}) = 0 \Rightarrow 4a + 3b = 0$$

$$\Rightarrow a = -\frac{3b}{4} \Rightarrow \frac{a}{+3} = \frac{b}{-4} = \lambda$$

$$\Rightarrow a = +3\lambda, b = -4\lambda \quad \dots(i)$$

$$\therefore \vec{c} = \lambda(3\hat{i} - 4\hat{j})$$

Now, let  $\vec{a} = x\hat{i} + y\hat{j}$  be the required vectors.

$$\text{Projection of } \vec{a} \text{ along } \vec{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \Rightarrow \frac{4x + 3y}{\sqrt{4^2 + 3^2}} = 1$$

$$\Rightarrow 4x + 3y = 5 \quad \dots(ii)$$

Also, projection of  $\vec{a}$  along  $\vec{c} = 2$

$$\Rightarrow \frac{\vec{a} \cdot \vec{c}}{|\vec{c}|} = 2 \Rightarrow \frac{3\lambda x - 4\lambda y}{\sqrt{(3\lambda)^2 + (-4\lambda)^2}} = 2$$

$$\Rightarrow 3\lambda x - 4\lambda y = 10\lambda \Rightarrow 3x - 4y = 10 \quad \dots(iii)$$

Solving (ii) and (iii), we get  $x = 2, y = -1$

$\therefore$  The required vector is  $2\hat{i} - \hat{j}$

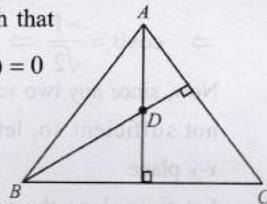
12. Given that the position vectors of points  $A, B, C$  and  $D$  are  $\vec{a}, \vec{b}, \vec{c}, \vec{d}$  respectively, such that

$$(\vec{a} - \vec{d}) \cdot (\vec{b} - \vec{c}) = (\vec{b} - \vec{d}) \cdot (\vec{c} - \vec{a}) = 0$$

$$\Rightarrow \overline{DA} \cdot \overline{CB} = \overline{DB} \cdot \overline{AC} = 0$$

$$\Rightarrow \overline{DA} \perp \overline{CB} \text{ and } \overline{DB} \perp \overline{AC}$$

Clearly  $D$  is orthocentre of  $\Delta ABC$



13. Given that  $|\vec{A}| = 3; |\vec{B}| = 4; |\vec{C}| = 5$

$$\because \vec{A} \perp (\vec{B} + \vec{C}) \Rightarrow \vec{A} \cdot (\vec{B} + \vec{C}) = 0$$

$$\vec{B} \perp (\vec{C} + \vec{A}) \Rightarrow \vec{B} \cdot (\vec{C} + \vec{A}) = 0$$

$$\vec{C} \perp (\vec{A} + \vec{B}) \Rightarrow \vec{C} \cdot (\vec{A} + \vec{B}) = 0$$

$$\text{Now, } |\vec{A} + \vec{B} + \vec{C}|^2 = (\vec{A} + \vec{B} + \vec{C}) \cdot (\vec{A} + \vec{B} + \vec{C})$$

$$= |\vec{A}|^2 + \vec{A} \cdot (\vec{B} + \vec{C}) + |\vec{B}|^2 + \vec{B} \cdot (\vec{C} + \vec{A}) + |\vec{C}|^2 + \vec{C} \cdot (\vec{A} + \vec{B})$$

$$= |\vec{A}|^2 + |\vec{B}|^2 + |\vec{C}|^2 + 0$$

$$= 9 + 16 + 25 = 50 \therefore |\vec{A} + \vec{B} + \vec{C}| = 5\sqrt{2}$$

14. (a, c, d)  $|\vec{a}| = \frac{1}{3}\sqrt{(4+4+1)} = 1 \Rightarrow |\vec{a}| = 1$

$\therefore$  It is unit vector

Let  $\vec{b} = 2\hat{i} - 4\hat{j} + 3\hat{k}$  then

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \frac{5}{\sqrt{29}} \Rightarrow \theta \neq \frac{\pi}{3}$$

$$\text{Let } \vec{c} = -\hat{i} + \hat{j} - \frac{1}{2}\hat{k} = \frac{-3}{2}\vec{a} \Rightarrow \vec{c} \parallel \vec{a}$$

Let  $\vec{d} = 3\hat{i} + 2\hat{j} + 2\hat{k}$  then  $\vec{a} \cdot \vec{d} = 0 \Rightarrow \vec{a} \perp \vec{d}$

15. (a, c) Any vector in the plane of  $\vec{b}$  and  $\vec{c}$  is  $\vec{u} = \vec{b} + \lambda\vec{c}$

$$= (\hat{i} + 2\hat{j} - \hat{k}) + \lambda(\hat{i} + \hat{j} - 2\hat{k})$$

$$= (1 + \lambda)\hat{i} + (2 + \lambda)\hat{j} - (1 + 2\lambda)\hat{k}$$

Given that magnitude of projection of  $\vec{u}$  on  $\vec{a}$  is  $\sqrt{\frac{2}{3}}$

$$\Rightarrow \sqrt{\frac{2}{3}} = \frac{|\vec{u} \cdot \vec{a}|}{|\vec{a}|} \Rightarrow \sqrt{\frac{2}{3}} = \frac{|2(1 + \lambda) - (2 + \lambda) - (1 + 2\lambda)|}{\sqrt{6}}$$

$$\Rightarrow |-\lambda - 1| = 2 \Rightarrow \lambda + 1 = 2 \text{ or } \lambda + 1 = -2$$

$$\Rightarrow \lambda = 1 \text{ or } \lambda = -3$$

$\therefore$  The required vector is either,

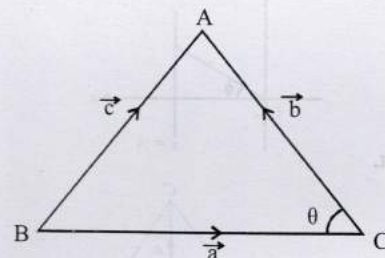
$$2\hat{i} + 3\hat{j} - 3\hat{k} \text{ or } -2\hat{i} - \hat{j} + 5\hat{k}$$

16.  $\vec{A} \rightarrow \vec{q}, \vec{B} \rightarrow \vec{p}, \vec{C} \rightarrow \vec{s}, \vec{D} \rightarrow \vec{t}$

A.  $|\vec{a}| = \sqrt{1+3} = 2$

$$|\vec{b}| = \sqrt{1+3} = 2$$

$$|\vec{c}| = 2\sqrt{3}$$



Using cosine formula

$$\cos C = \frac{|\vec{a}|^2 + |\vec{b}|^2 - |\vec{c}|^2}{2|\vec{a}||\vec{b}|} = \frac{4 + 4 - 12}{2 \times 2 \times 2} = \frac{1}{2}$$

$$\Rightarrow \cos \theta = -\frac{1}{2} \Rightarrow \theta = \frac{2\pi}{3}$$

$\Rightarrow \vec{A} \rightarrow \vec{q}$

B.  $\int_a^b (f(x) - 3x) dx = a^2 - b^2$

$$\Rightarrow \int_a^b f(x) dx + \frac{3}{2}[-b^2 + a^2] = a^2 - b^2$$

$$\Rightarrow \int_a^b f(x) dx = -\frac{1}{2}(a^2 - b^2) = \int_a^b \frac{1}{2}x dx$$

$$\Rightarrow f(x) = x \Rightarrow f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$$

$\Rightarrow B \rightarrow p$

$$C. \frac{\pi^2}{\ln 3} \int_{\frac{7}{6}}^{\frac{5}{6}} \sec(\pi x) dx = \frac{\pi^2}{\pi \ln 3} \left[ \ln |\sec \pi x + \tan \pi x| \right]_{\frac{7}{6}}^{\frac{5}{6}}$$

$$= \frac{\pi}{\ln 3} \left[ \ln \left| \sec \frac{5\pi}{6} + \tan \frac{5\pi}{6} \right| - \ln \left| \sec \frac{7\pi}{6} + \tan \frac{7\pi}{6} \right| \right]$$

$$= \frac{\pi}{\ln 3} \left[ \ln \left| -\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right| - \ln \left| -\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} \right| \right] = \frac{\pi}{\ln 3} \ln 3 = \pi$$

$\therefore C \rightarrow s$

D. For  $|z| = 1$  and  $z \neq 1$ . Let  $z = \cos \theta + i \sin \theta$

$$\text{Then } 1 - z = 1 - \cos \theta - i \sin \theta = 2 \sin^2 \frac{\theta}{2} - 2i \sin \frac{\theta}{2} \cos \frac{\theta}{2}$$

$$\text{or } 1 - z = 2 \sin \frac{\theta}{2} \left[ \sin \frac{\theta}{2} - i \cos \frac{\theta}{2} \right]$$

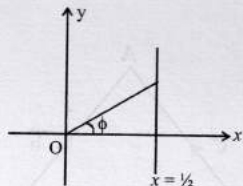
$$\therefore \frac{1}{1-z} = \frac{1}{2} \left[ 1 + i \cot \frac{\theta}{2} \right]$$

Clear that real part of  $\frac{1}{1-z}$  is always  $\frac{1}{2}$

$\therefore$  Locus of  $\frac{1}{1-z}$  is  $x = \frac{1}{2}$

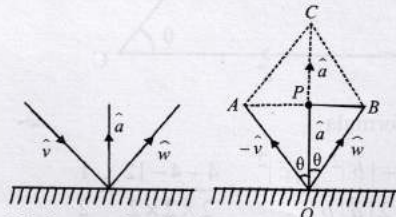
$\left| \text{Arg} \left( \frac{1}{1-z} \right) \right|$  is maximum when value of  $\phi$  approaches to

$\frac{\pi}{2}$  but will not be attained.



$\therefore D \rightarrow t$

17.



We know that incident ray, reflected ray and normal lie in a same plane, and angle of incidence = angle of reflection.

Therefore unit vector  $\hat{a}$  will be along the angle bisector of  $\hat{w}$  and  $-\hat{v}$ ,

$$\text{i.e., } \hat{a} = \frac{\hat{w} + (-\hat{v})}{|\hat{w} - \hat{v}|} \quad \dots(1)$$

$\therefore$  Angle bisector will along a vector dividing in same ratio as the ratio of the sides forming that angle.]

Since  $|\hat{w} - \hat{v}| = OC = 2OP = 2|\hat{w}| \cos \theta = 2 \cos \theta$

Substituting this value in equation (1) we get

$$\hat{a} = \frac{\hat{w} - \hat{v}}{2 \cos \theta}$$

$$\therefore \hat{w} = \hat{v} + (2 \cos \theta) \hat{a} = \hat{v} - 2(\hat{a} \cdot \hat{v}) \hat{a} \quad [\because \hat{a} \cdot \hat{v} = -\cos \theta]$$

18. We have  $\vec{A}(t)$  is parallel to  $\vec{B}(t)$  for some  $t \in [0, 1]$  if and only if

$$\frac{f_1(t)}{g_1(t)} = \frac{f_2(t)}{g_2(t)} \text{ for some } t \in [0, 1]$$

$$\Rightarrow f_1(t) \cdot g_2(t) = f_2(t) \cdot g_1(t) \text{ for some } t \in [0, 1]$$

$$\text{Let } h(t) = f_1(t) \cdot g_2(t) - f_2(t) \cdot g_1(t)$$

$$h(0) = f_1(0) \cdot g_2(0) - f_2(0) \cdot g_1(0)$$

$$= 2 \times 2 - 3 \times 3 = -5 < 0$$

$$h(1) = f_1(1) \cdot g_2(1) - f_2(1) \cdot g_1(1)$$

$$= 6 \times 6 - 2 \times 2 = 32 > 0$$

Since  $h(t)$  is a continuous function, and  $h(0) \cdot h(1) < 0$

$\Rightarrow$  There is some  $t \in [0, 1]$  for which  $h(t) = 0$  i.e.,  $\vec{A}(t)$  and  $\vec{B}(t)$  are parallel vectors for this  $t$ .

19. From the given data, we get

$$\vec{v}_1 \cdot \vec{v}_1 = 4 \Rightarrow |\vec{v}_1| = 2$$

$$\vec{v}_2 \cdot \vec{v}_2 = 2 \Rightarrow |\vec{v}_2| = \sqrt{2}$$

$$\vec{v}_3 \cdot \vec{v}_3 = 29 \Rightarrow |\vec{v}_3| = \sqrt{29}$$

Let  $\theta$  is the angle between  $\vec{v}_1$  and  $\vec{v}_2$

$$\therefore \vec{v}_1 \cdot \vec{v}_2 = -2 \Rightarrow |\vec{v}_1| |\vec{v}_2| \cos \theta = -2$$

$$\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \Rightarrow \theta = 135^\circ$$

Now, since any two vectors are always coplanar and data is not sufficient so, let us suppose that  $\vec{v}_1$  and  $\vec{v}_2$  are in  $x$ - $y$  plane.

Let  $\vec{v}_1$  is along the positive direction of  $x$ -axis

$$\text{then } \vec{v}_1 = 2\hat{i}. \quad [\because |\vec{v}_1| = 2]$$

$$\therefore \vec{v}_2 \text{ makes an angle } 135^\circ \text{ with } \vec{v}_1 \text{ and } |\vec{v}_2| = \sqrt{2},$$

$$\therefore \vec{v}_2 = -\hat{i} \pm \hat{j}$$

$$\text{Let } \vec{v}_3 = \alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$$

$$\therefore \vec{v}_3 \cdot \vec{v}_1 = 6 \Rightarrow 2\alpha = 6 \Rightarrow \alpha = 3$$

$$\text{and } \vec{v}_3 \cdot \vec{v}_2 = -5 \Rightarrow -\alpha + \beta = -5 \Rightarrow \beta = \pm 2$$

$$\text{Also } |\vec{v}_3| = \sqrt{29} \Rightarrow \alpha^2 + \beta^2 + \gamma^2 = 29 \Rightarrow \gamma = \pm 4$$

$$\text{Hence } \vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$$

$$\text{Thus, } \vec{v}_1 = 2\hat{i}; \vec{v}_2 = -\hat{i} \pm \hat{j}; \vec{v}_3 = 3\hat{i} \pm 2\hat{j} \pm 4\hat{k}$$

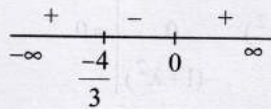
are some possible answers.

20.  $\vec{a} \cdot \vec{b} = cx^2 - 12 - 6cx$

$$\text{Since } \vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

Given that angle between  $\vec{a}$  and  $\vec{b}$  is obtuse, therefore

$$\begin{aligned} \cos \theta < 0 &\Rightarrow \vec{a} \cdot \vec{b} < 0 \\ &\Rightarrow cx^2 - 12 - 6cx < 0 \\ &\Rightarrow cx^2 - 6cx - 12 < 0, \forall x \in R \quad \dots(i) \\ \text{Clearly above condition is satisfied if } c &= 0 \\ \text{And } c < 0 \text{ and } D < 0 \\ &\Rightarrow 36c^2 + 48c < 0 \Rightarrow 12c(3c + 4) < 0 \end{aligned}$$



$$\therefore \frac{-4}{3} < c \leq 0.$$

21. Let with respect to  $O$ , position vectors of points  $A, B, C, D, E, F$  be  $\vec{a}, \vec{b}, \vec{c}, \vec{d}, \vec{e}, \vec{f}$ .

Let perpendiculars from  $A$  to  $EF$  and from  $B$  to  $DF$  intersect each other at  $H$ . Let position vector of  $H$  be  $\vec{r}$ . We join  $CH$ . Know we have to prove that  $CH$  is perpendicular to  $DE$ .

$$\text{Given that } OD \perp BC \Rightarrow \vec{d} \cdot (\vec{c} - \vec{b}) = 0$$

$$\Rightarrow \vec{d} \cdot \vec{b} = \vec{d} \cdot \vec{c} \quad \dots(i)$$

$$OE \perp AC \Rightarrow \vec{e} \cdot (\vec{c} - \vec{a}) = 0 \Rightarrow \vec{e} \cdot \vec{c} = \vec{e} \cdot \vec{a} \quad \dots(ii)$$

$$\text{And } OF \perp AB \Rightarrow \vec{f} \cdot (\vec{b} - \vec{a}) = 0 \Rightarrow \vec{f} \cdot \vec{a} = \vec{f} \cdot \vec{b} \quad \dots(iii)$$

$$\text{Also given that } AH \perp EF \Rightarrow (\vec{r} - \vec{a}) \cdot (\vec{f} - \vec{e}) = 0$$

$$\Rightarrow \vec{r} \cdot \vec{f} - \vec{r} \cdot \vec{e} - \vec{a} \cdot \vec{f} + \vec{a} \cdot \vec{e} = 0 \quad \dots(iv)$$

$$\text{and } BH \perp FD \Rightarrow (\vec{r} - \vec{b}) \cdot (\vec{d} - \vec{f}) = 0$$

$$\Rightarrow \vec{r} \cdot \vec{d} - \vec{r} \cdot \vec{f} - \vec{b} \cdot \vec{d} + \vec{b} \cdot \vec{f} = 0 \quad \dots(v)$$

Adding (iv) and (v), we get

$$\vec{r} \cdot \vec{d} - \vec{r} \cdot \vec{e} - \vec{a} \cdot \vec{f} + \vec{a} \cdot \vec{e} - \vec{b} \cdot \vec{d} + \vec{b} \cdot \vec{f} = 0$$

$$\Rightarrow \vec{r} \cdot \vec{d} - \vec{r} \cdot \vec{e} + \vec{e} \cdot \vec{c} - \vec{d} \cdot \vec{c} = 0$$

(using (i), (ii) and (iii))

$$\Rightarrow \vec{r} \cdot (\vec{d} - \vec{e}) - \vec{c} \cdot (\vec{d} - \vec{e}) = 0 \Rightarrow (\vec{r} - \vec{c}) \cdot (\vec{d} - \vec{e}) = 0$$

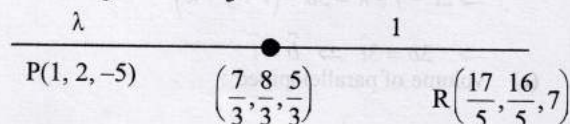
$$\Rightarrow \vec{CH} \cdot \vec{ED} = 0 \Rightarrow CH \perp ED \quad \text{Hence Proved.}$$



**Topic-3: Vector or Cross Product of two vectors, Scalar & Vector Triple Product**

1. (b)  $P(1, 2, -5), Q(3, 6, 3), R\left(\frac{17}{5}, \frac{16}{5}, 7\right), S(2, 1, 1)$

$$\text{Now, } \frac{\vec{b} + 2\vec{d}}{3} = \frac{7\hat{i} + 8\hat{j} + 5\hat{k}}{3}$$



$$\begin{aligned} \frac{7}{3} &= \frac{17\lambda}{5} + 1; \frac{8}{3} = \frac{16\lambda}{5} + 2; \frac{5}{3} = \frac{7\lambda - 5}{\lambda + 1} \\ &\Rightarrow \lambda = \frac{5}{4}; \lambda = \frac{5}{4}; \lambda = \frac{5}{4} \end{aligned}$$

2. (c) Given that  $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$

$$\Rightarrow (\vec{a} + \vec{b}) \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = \vec{0}$$

$$\text{But } \vec{a} + \vec{b} \neq \vec{0} \text{ and } 2\hat{i} + 3\hat{j} + 4\hat{k} \neq \vec{0}$$

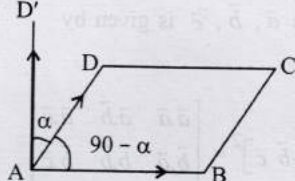
$$\therefore (\vec{a} + \vec{b}) \parallel (2\hat{i} + 3\hat{j} + 4\hat{k}).$$

$$\text{Let } \vec{a} + \vec{b} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\text{Also given that } |\vec{a} + \vec{b}| = \sqrt{29} \Rightarrow \lambda = \pm 1$$

$$\therefore \vec{a} + \vec{b} = \pm(2\hat{i} + 3\hat{j} + 4\hat{k})$$

$$\text{So, } (\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k}) = \pm 4$$

3. (b) 

$$\vec{AB} \times \vec{AD} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 10 & 11 \\ -1 & 2 & 2 \end{vmatrix} = -2\hat{i} - 15\hat{j} + 14\hat{k}$$

$$\therefore |\vec{AB} \times \vec{AD}| = \sqrt{4 + 225 + 196} = \sqrt{425}$$

$$|\vec{AB}| = \sqrt{4 + 100 + 121} = \sqrt{225} = 15$$

$$|\vec{AD}| = \sqrt{1 + 4 + 4} = 3$$

$$\therefore \sin(90 - \alpha) = \frac{|\vec{AB} \times \vec{AD}|}{|\vec{AB}| |\vec{AD}|}$$

$$\therefore \sin(90 - \alpha) = \frac{\sqrt{425}}{15 \times 3} = \frac{\sqrt{17}}{9} \Rightarrow \cos \alpha = \frac{\sqrt{17}}{9}$$

4. (c) Given:  $\vec{a}, \vec{b}, \vec{c}$  and  $\vec{d}$  are unit vectors,

$$\text{Let } \vec{a} \times \vec{b} = (\sin \alpha) \vec{n}_1 \text{ and } \vec{c} \times \vec{d} = (\sin \beta) \vec{n}_2$$

where  $\vec{n}_1$  and  $\vec{n}_2$  are unit normal vectors

$$\text{then } (\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = 1$$

$$\Rightarrow (\sin \alpha) \vec{n}_1 \cdot (\sin \beta) \vec{n}_2 = 1$$

$$\Rightarrow \sin \alpha \sin \beta \vec{n}_1 \cdot \vec{n}_2 = 1 \Rightarrow \sin \alpha \sin \beta \cos \gamma = 1$$

where  $\gamma$  is the angle between  $\vec{n}_1$  and  $\vec{n}_2$ .

$$\Rightarrow \alpha = \frac{\pi}{2}, \beta = \frac{\pi}{2} \text{ and } \gamma = 0^\circ$$

$$\text{If } \gamma = 0^\circ \Rightarrow \vec{a} \times \vec{b} \parallel \vec{c} \times \vec{d}$$

$$\text{Let } \vec{a} \times \vec{b} = \lambda(\vec{c} \times \vec{d}) \Rightarrow (\vec{a} \times \vec{b}) \cdot \vec{c} = \lambda(\vec{c} \times \vec{d}) \cdot \vec{c} = 0$$

$$\text{and } (\vec{a} \times \vec{b}) \cdot \vec{d} = \lambda(\vec{c} \times \vec{d}) \cdot \vec{d} = 0$$

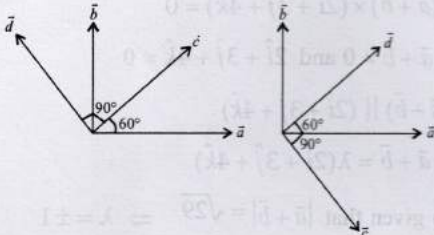
$$[\vec{a} \vec{b} \vec{c}] = [\vec{a} \vec{b} \vec{d}] = 0$$

∴  $\vec{a}, \vec{b}, \vec{c}$  are coplanar and  $\vec{a}, \vec{b}, \vec{d}$  are coplanar

⇒  $\vec{a}, \vec{b}, \vec{c}, \vec{d}$  are coplanar

and  $\alpha = 90^\circ \Rightarrow \vec{a} \perp \vec{b}$  and  $\beta = 90^\circ \Rightarrow \vec{c} \perp \vec{d}$

But angle between  $\vec{a}$  and  $\vec{c}$  is  $\pi/3$  ( $\because \vec{a} \cdot \vec{c} = \frac{1}{2}$ )



The possible cases are shown in figures and in any case  $\vec{b}$  and  $\vec{d}$  are non-parallel vectors.

5. (a) The volume of a parallelepiped with coterminus edges as the vectors  $\vec{a}, \vec{b}, \vec{c}$  is given by

$$V = [\vec{a} \vec{b} \vec{c}]$$

$$\text{We have } [\vec{a} \vec{b} \vec{c}]^2 = \begin{vmatrix} \vec{a} \cdot \vec{a} & \vec{a} \cdot \vec{b} & \vec{a} \cdot \vec{c} \\ \vec{b} \cdot \vec{a} & \vec{b} \cdot \vec{b} & \vec{b} \cdot \vec{c} \\ \vec{c} \cdot \vec{a} & \vec{c} \cdot \vec{b} & \vec{c} \cdot \vec{c} \end{vmatrix}$$

$$\Rightarrow V^2 = \begin{vmatrix} 1 & 1/2 & 1/2 \\ 1/2 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{vmatrix} = \frac{1}{2} \Rightarrow V = \frac{1}{\sqrt{2}}$$

6. (b) Since,  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$

$$\Rightarrow \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$$

( $\because \vec{a} \times \vec{a} = \vec{0}$  and  $\vec{a} \times \vec{c} = -\vec{c} \times \vec{a}$ )

Similarly,  $\vec{b} \times \vec{c} = \vec{c} \times \vec{a}$

$$\therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$$

Also since  $\vec{a}, \vec{b}, \vec{c}$  are non-parallel and unit vector (these form an equilateral  $\Delta$ ).

$$\therefore \vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a} \neq \vec{0}$$

7. (c) Since given three vectors are coplanar.

$$\therefore \begin{vmatrix} -\lambda^2 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

Applying,  $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 2-\lambda^2 & 2-\lambda^2 & 2-\lambda^2 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda^2) \begin{vmatrix} 1 & 1 & 1 \\ 1 & -\lambda^2 & 1 \\ 1 & 1 & -\lambda^2 \end{vmatrix} = 0$$

Applying  $R_2 = R_2 - R_1$  and  $R_3 = R_3 - R_1$

$$\Rightarrow (2-\lambda^2) \begin{vmatrix} 1 & 1 & 1 \\ 0 & -(1+\lambda^2) & 0 \\ 0 & 0 & -(1+\lambda^2) \end{vmatrix} = 0$$

$$\Rightarrow (2-\lambda^2)(1+\lambda^2)^2 = 0 \Rightarrow \lambda = \pm\sqrt{2} \quad [\because 1+\lambda^2 \neq 0]$$

∴ Two real solutions.

8. (a) We know that a vector in the plane of  $\vec{a}$  and  $\vec{b}$  is

$$\vec{u} = \vec{a} + \lambda\vec{b} = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (1+\lambda)\hat{k}$$

Given that projection of  $\vec{u}$  on  $\vec{c} = \frac{1}{\sqrt{3}}$

$$\Rightarrow \frac{\vec{u} \cdot \vec{c}}{|\vec{c}|} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \frac{\vec{u} \cdot \vec{c}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \vec{u} \cdot \vec{c} = 1 \Rightarrow |1+\lambda+2-\lambda-1-\lambda| = 1$$

$$\Rightarrow |2-\lambda| = 1 \Rightarrow \lambda = 1 \text{ or } 3$$

$$\Rightarrow \vec{u} = 2\hat{i} + \hat{j} + 2\hat{k} \text{ or } 4\hat{i} - \hat{j} + 4\hat{k}$$

9. (c) We know that any vector which is coplanar to  $\vec{a}$  and  $\vec{b}$  can be written as

$$\vec{r} = \vec{a} + \lambda\vec{b} = (\hat{i} - \hat{j} + \hat{k}) + \lambda(2\hat{i} + \hat{j} + \hat{k})$$

$$\vec{r} = (1+2\lambda)\hat{i} + (-1+\lambda)\hat{j} + (1+\lambda)\hat{k}$$

Since  $\vec{r}$  is orthogonal to  $5\hat{i} + 2\hat{j} + 6\hat{k}$

$$\Rightarrow \vec{r} \cdot (5\hat{i} + 2\hat{j} + 6\hat{k}) = 0$$

$$\Rightarrow 5(1+2\lambda) + 2(-1+\lambda) + 6(1+\lambda) = 0$$

$$\Rightarrow 9 + 18\lambda = 0 \Rightarrow \lambda = -\frac{1}{2}$$

$$\therefore \vec{r} \text{ is } 3\hat{j} - \hat{k}$$

$$\therefore \vec{r} = \frac{3\hat{j} - \hat{k}}{\sqrt{10}}$$

10. (c) We have  $(\vec{a} \times \vec{b}) \times \vec{a} = (\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a}$

$$\therefore (\hat{j} - \hat{k}) \times (\hat{i} + \hat{j} + \hat{k}) = (\sqrt{3})^2(\vec{b}) - (\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow 2\hat{i} - \hat{j} - \hat{k} = 3\vec{b} - (\hat{i} + \hat{j} + \hat{k})$$

$$\Rightarrow 3\vec{b} = 3\hat{i} \Rightarrow \vec{b} = \hat{i}$$

11. (c) Volume of parallelepiped :

$$V = \begin{vmatrix} 1 & a & 1 \\ 0 & 1 & a \\ a & 0 & 1 \end{vmatrix}$$

$$= 1(1-0) - a(0-a^2) + 1(0-a) = 1 + a^3 - a$$

$$\frac{dV}{da} = 3a^2 - 1$$

For max. and min.

$$\Rightarrow 3a^2 - 1 = 0 \Rightarrow a = \pm \frac{1}{\sqrt{3}}$$

$$\frac{d^2V}{da^2} = 6a \Rightarrow \left(\frac{d^2V}{da^2}\right)_{a=\frac{1}{\sqrt{3}}} = \frac{6}{\sqrt{3}} > 0$$

\(\therefore\) V is minimum at  $a = \frac{1}{\sqrt{3}}$

12. (c) Given that  $\vec{V} = 2\hat{i} + \hat{j} - \hat{k}$  and  $\vec{W} = \hat{i} + 3\hat{k}$  and  $\vec{U}$  is a unit vector  $\therefore |\vec{U}| = 1$

$$\vec{V} \times \vec{W} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = 3\hat{i} - 7\hat{j} - \hat{k}$$

$$\text{Now, } [\vec{U} \vec{V} \vec{W}] = \vec{U} \cdot (\vec{V} \times \vec{W})$$

$$= \vec{U} \cdot (3\hat{i} - 7\hat{j} - \hat{k}) = |\vec{U}| \sqrt{3^2 + 7^2 + 1^2} \cos \theta$$

$$= \sqrt{59} \cos \theta$$

which is max. when  $\cos \theta = 1$

\(\therefore\) Max. value of  $[\vec{U} \vec{V} \vec{W}] = \sqrt{59}$

13. (c)  $[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}$
- $$= 1(1+x-y-x+x^2) - 1(x^2-y) = 1$$
- \(\therefore\) It neither depends on x nor on y.

14. (a) Since  $\vec{a}, \vec{b}, \vec{c}$  are unit coplanar vectors, then their linear combination  $2\vec{a} - \vec{b}, 2\vec{b} - \vec{c}$  and  $2\vec{c} - \vec{a}$  are also coplanar vectors.

$$\text{Thus, } [2\vec{a} - \vec{b} \quad 2\vec{b} - \vec{c} \quad 2\vec{c} - \vec{a}] = 0$$

15. (a) \(\because\)  $P_1$  is the plane determined by vectors  $\vec{a}$  and  $\vec{b}$   
Let  $\vec{n}_1$  be normal vector of  $P_1$  then  $\vec{n}_1 = \vec{a} \times \vec{b}$   
Now,  $P_2$  is the plane determined by vectors  $\vec{c}$  and  $\vec{d}$   
Let  $\vec{n}_2$  be normal vector of  $P_2$  then  $\vec{n}_2 = \vec{c} \times \vec{d}$   
Now given that  $(\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) = 0$

$$\therefore \vec{n}_1 \times \vec{n}_2 = 0 \Rightarrow \vec{n}_1 \parallel \vec{n}_2$$

and hence the planes will also be parallel to each other.

Thus angle between the planes = 0.

16. (b) Given that  $\vec{a} + \vec{b} + \vec{c} = 0$  (by triangle law)

$$\therefore \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{a} \times 0 = \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$$

$$\Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a} \quad [\because \vec{a} \times \vec{a} = 0 \text{ and } \vec{a} \times \vec{c} = -\vec{c} \times \vec{a}]$$

Similarly,  $\vec{a} \times \vec{b} = \vec{b} \times \vec{c}$ ;

Therefore  $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ .

17. (a) Let  $\vec{c} = x\hat{i} + y\hat{j} + z\hat{k}$

Given that  $\vec{a}, \vec{b}$  and  $\vec{c}$  are coplanar

$$\therefore [\vec{a} \vec{b} \vec{c}] = 0$$

$$\Rightarrow \begin{vmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ x & y & z \end{vmatrix} = 0 \Rightarrow x - y - z = 0 \quad \dots(i)$$

\(\because\)  $\vec{c}$  is  $\perp$  to  $\vec{a}$ ,  $\therefore \vec{a} \cdot \vec{c} = 0$

$$2x + y + z = 0 \quad \dots(ii)$$

from (i) and (ii)

$$\frac{x}{0} = \frac{y}{3} = \frac{z}{-3} = \lambda \Rightarrow x = 0, y = 3\lambda \text{ and } z = -3\lambda$$

But  $|\vec{c}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1$

$$\Rightarrow 9\lambda^2 + 9\lambda^2 = 1 \Rightarrow \lambda = \pm \frac{1}{3\sqrt{2}}$$

$$\therefore \vec{c} = \pm \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$$

Thus, we have  $\vec{c} = \frac{1}{\sqrt{2}}(-\hat{j} + \hat{k})$

18. (b)  $|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}| |\vec{c}| \sin 30^\circ$

$$= \frac{1}{2} |\vec{a} \times \vec{b}| |\vec{c}| \quad \dots(i)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 1 & 1 & 0 \end{vmatrix} = 2\hat{i} - 2\hat{j} + \hat{k}$$

$$\Rightarrow |\vec{a} \times \vec{b}| = \sqrt{9} = 3$$

Also given that  $|\vec{c} - \vec{a}| = 2\sqrt{2} \Rightarrow |\vec{c} - \vec{a}|^2 = 8$

$$\Rightarrow |\vec{c}|^2 - \vec{c} \cdot \vec{a} - \vec{a} \cdot \vec{c} + |\vec{a}|^2 = 8$$

\(\because\)  $|\vec{a}| = 3$  and  $\vec{a} \cdot \vec{c} = |\vec{c}|$ , we get

$$|\vec{c}|^2 - 2|\vec{c}| + 1 = 0 \quad (|\vec{c}| - 1)^2 = 0 \Rightarrow |\vec{c}| = 1$$

Substituting values of  $|\vec{a} \times \vec{b}|$  and  $|\vec{c}|$  in (i), we get

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = \frac{1}{2} \times 3 \times 1 = \frac{3}{2}$$

19. (d)  $(\vec{a} + \vec{b} + \vec{c}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} + \vec{c})]$
- $$= (\vec{a} + \vec{b} + \vec{c}) \cdot [\vec{a} \times \vec{a} + \vec{a} \times \vec{c} + \vec{b} \times \vec{a} + \vec{b} \times \vec{c}]$$
- $$= (\vec{a} + \vec{b} + \vec{c}) \cdot [\vec{a} \times \vec{c} + \vec{b} \times \vec{a} + \vec{b} \times \vec{c}] \quad [\because \vec{a} \times \vec{a} = 0]$$
- $$= \vec{a} \cdot (\vec{a} \times \vec{c}) + \vec{a} \cdot (\vec{b} \times \vec{a}) + \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{a} \times \vec{c})$$



$$+\vec{b} \cdot (\vec{b} \times \vec{a}) + \vec{b} \cdot (\vec{b} \times \vec{c}) + \vec{c} \cdot (\vec{a} \times \vec{c}) + \vec{c} \cdot (\vec{b} \times \vec{a}) + \vec{c} \cdot (\vec{b} \times \vec{c})$$

$$= [\vec{a}\vec{b}\vec{c}] - [\vec{a}\vec{b}\vec{c}] - [\vec{a}\vec{b}\vec{c}] = -[\vec{a}\vec{b}\vec{c}] \quad [\because [\vec{a}\vec{a}\vec{b}] = 0]$$

20. (a) Since  $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\vec{b} + \vec{c}}{\sqrt{2}}$

$$\therefore (\vec{a}\vec{c})\vec{b} - (\vec{a}\vec{b})\vec{c} = \frac{1}{\sqrt{2}}\vec{b} + \frac{1}{\sqrt{2}}\vec{c}$$

[ $\because \vec{b}$  and  $\vec{c}$  are non-coplanar]

On comparing both sides, we get

$$\Rightarrow \vec{a}\vec{c} = \frac{1}{\sqrt{2}} \text{ and } \vec{a}\vec{b} = -\frac{1}{\sqrt{2}} \Rightarrow \cos \theta = -\frac{1}{\sqrt{2}}$$

$$\Rightarrow \cos \theta = \cos \frac{3\pi}{4} \Rightarrow \theta = 3\pi/4$$

21. (a) Let  $\vec{d} = x\hat{i} + y\hat{j} + z\hat{k}$

Given that  $|\vec{d}| = 1 \Rightarrow x^2 + y^2 + z^2 = 1$  ....(i)

and  $\vec{a}\vec{d} = 0$

$$\Rightarrow x - y = 0 \Rightarrow x = y$$
 ....(ii)

Now,  $[\vec{b}\vec{c}\vec{d}] = 0 \Rightarrow \begin{vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ x & y & z \end{vmatrix} = 0$

$$\Rightarrow x + y + z = 0$$

$$\Rightarrow 2x + z = 0 \quad \text{(from (i))}$$

$$\Rightarrow z = -2x$$
 ....(iii)

From (i), (ii) and (iii)

$$x^2 + x^2 + 4x^2 = 1 \Rightarrow x = \pm \frac{1}{\sqrt{6}}$$

$$\therefore \vec{d} = \pm \left( \frac{1}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} - \frac{2}{\sqrt{6}}\hat{k} \right) = \pm \left( \frac{\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{6}} \right)$$

22. (b) Given that a, b, c are distinct non negative numbers and the vectors  $a\hat{i} + a\hat{j} + c\hat{k}$ ,  $\hat{i} + \hat{k}$  and  $c\hat{i} + c\hat{j} + b\hat{k}$  are coplanar.

$$\therefore \begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Applying  $C_3 \rightarrow C_3 - C_1$

$$\Rightarrow \begin{vmatrix} a & a & c-a \\ 1 & 0 & 0 \\ c & c & b-c \end{vmatrix} = 0$$

Expanding along  $R_2$ , we get

$$= [c(c-a) - a(b-c)] = 0$$

$$\Rightarrow c^2 - ac - ab + ac = 0$$

$$\Rightarrow c^2 = ab \Rightarrow a, c, b \text{ are in G.P.}$$

$\therefore c$  is the G.M. of  $a$  and  $b$ .

23. (d) Given that  $\vec{a}, \vec{b}, \vec{c}$  are non coplanar

$$\therefore [\vec{a}\vec{b}\vec{c}] \neq 0$$

And  $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}$ ,  $\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$ ,  $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$

Now,  $(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r}$

$$= (\vec{a} + \vec{b}) \cdot \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]} + (\vec{b} + \vec{c}) \cdot \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]} + (\vec{c} + \vec{a}) \cdot \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$$

$$= \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{[\vec{a}\vec{b}\vec{c}]} + \frac{\vec{b} \cdot (\vec{c} \times \vec{a})}{[\vec{a}\vec{b}\vec{c}]} + \frac{\vec{c} \cdot (\vec{a} \times \vec{b})}{[\vec{a}\vec{b}\vec{c}]}$$

[ $\because \vec{b}\vec{b} \times \vec{c} = \vec{c}\vec{c} \times \vec{a} = \vec{a}\vec{a} \times \vec{b} = 0$ ]

$$= \frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} + \frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} + \frac{[\vec{a}\vec{b}\vec{c}]}{[\vec{a}\vec{b}\vec{c}]} = 1 + 1 + 1 = 3$$

24. (d) Volume of parallelopiped =  $[\vec{OA}\vec{OB}\vec{OC}]$

$$= \begin{vmatrix} 2 & -2 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} = 2(-1) + 2(-1+3) = 2$$

25. (d)  $|(\vec{a} \times \vec{b}) \cdot \vec{c}| = |\vec{a}| |\vec{b}| |\vec{c}|$

$$\Rightarrow |\hat{a}| |\hat{b}| |\sin \theta| |\hat{n} \cdot \hat{c}| = |\hat{a}| |\hat{b}| |\hat{c}|$$

where  $\theta$  is angle between  $\vec{a}$  and  $\vec{b}$ .

$$\Rightarrow |\sin \theta \hat{n} \cdot \hat{c}| = |\hat{c}| \Rightarrow |\hat{c}| |\sin \theta \cos \alpha| = |\hat{c}|$$

where  $\alpha$  is angle between  $\vec{c}$  and  $\hat{n}$ .

$$\Rightarrow |\sin \theta| = 1 \Rightarrow \theta = \frac{\pi}{2} \Rightarrow \vec{a} \cdot \vec{b} = 0$$

and  $|\cos \alpha| = 1 \Rightarrow \alpha = 0 \Rightarrow \vec{c} \parallel \hat{n} \Rightarrow \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = 0$

$$\Rightarrow \vec{a}\vec{b} = \vec{b}\vec{c} = \vec{c}\vec{a} = 0$$

26. (a)  $\vec{A} \cdot (\vec{B} + \vec{C}) \times (\vec{A} + \vec{B} + \vec{C})$

$$= \vec{A} \cdot [\vec{B} \times \vec{A} + \vec{B} \times \vec{B} + \vec{B} \times \vec{C} + \vec{C} \times \vec{A} + \vec{C} \times \vec{B} + \vec{C} \times \vec{C}]$$

( $\because \vec{A} \times \vec{A} = 0$ )

$$= \vec{A} \cdot (\vec{B} \times \vec{A} + \vec{B} \times \vec{C} + \vec{C} \times \vec{A} + \vec{C} \times \vec{B})$$

$$= \vec{A} \cdot \vec{B} \times \vec{A} + \vec{A} \cdot \vec{B} \times \vec{C} + \vec{A} \cdot \vec{C} \times \vec{A} + \vec{A} \cdot \vec{C} \times \vec{B}$$

( $\because \vec{A} \cdot (\vec{B} \times \vec{A}) = 0$ )

$$= 0 + [\vec{A}\vec{B}\vec{C}] + 0 + [\vec{A}\vec{C}\vec{B}]$$

$$= [\vec{A}\vec{B}\vec{C}] - [\vec{A}\vec{B}\vec{C}] \quad (\because [\vec{a}\vec{b}\vec{c}] = -[\vec{a}\vec{c}\vec{b}])$$

$$= 0$$

27. (7) Given that

$$|\vec{u}| = 1; |\vec{v}| = 1; \vec{u} \cdot \vec{v} \neq 0; \vec{u} \cdot \vec{w} = 1; \vec{v} \cdot \vec{w} = 1$$

and  $\vec{w} \cdot \vec{w} = |\vec{w}|^2 = 4 \Rightarrow |\vec{w}| = 2$

Volume of parallelepiped =  $[\vec{u} \ \vec{v} \ \vec{w}] = \sqrt{2}$

$$\Rightarrow [\vec{u} \ \vec{v} \ \vec{w}]^2 = \begin{vmatrix} \vec{u} \cdot \vec{u} & \vec{u} \cdot \vec{v} & \vec{u} \cdot \vec{w} \\ \vec{v} \cdot \vec{u} & \vec{v} \cdot \vec{v} & \vec{v} \cdot \vec{w} \\ \vec{w} \cdot \vec{u} & \vec{w} \cdot \vec{v} & \vec{w} \cdot \vec{w} \end{vmatrix} = 2$$

$$\Rightarrow \begin{vmatrix} 1 & \vec{u} \cdot \vec{v} & 1 \\ \vec{u} \cdot \vec{v} & 1 & 1 \\ 1 & 1 & 4 \end{vmatrix} = 2 \Rightarrow \vec{u} \cdot \vec{v} = \frac{1}{2}$$

Now,  $|3\vec{u} + 5\vec{v}|^2 = (3\vec{u} + 5\vec{v}) \cdot (3\vec{u} + 5\vec{v})$   
 $= 9|\vec{u}|^2 + 25|\vec{v}|^2 + 30\vec{u} \cdot \vec{v}$   
 $= 9 + 25 + 15 = 49$

$\therefore |3\vec{u} + 5\vec{v}| = \sqrt{49} = 7$

28. (3) Given that  $|\vec{a}| = |\vec{b}| = 1$ ,  $\vec{a} \cdot \vec{b} = 0$  and  $|\vec{c}| = 2$

$\vec{c}$  makes angle  $\alpha$  with both  $\vec{a}$  and  $\vec{b}$

Also,  $\vec{c} = x\vec{a} + y\vec{b} + \vec{a} \times \vec{b}$

$\vec{c} \cdot \vec{a} = |\vec{c}| |\vec{a}| \cos \alpha = 2 \cos \alpha \Rightarrow x = 2 \cos \alpha$

$\vec{c} \cdot \vec{b} = 2 \cos \alpha \Rightarrow y = 2 \cos \alpha$

$|\vec{c}|^2 = \vec{c} \cdot \vec{c} = (2 \cos \alpha)\vec{a} + (2 \cos \alpha)\vec{b} + \vec{a} \times \vec{b}$

$\Rightarrow (2)^2 = 4 \cos^2 \alpha + 4 \cos^2 \alpha + |\vec{a} \times \vec{b}|^2$

$\Rightarrow 4 = 8 \cos^2 \alpha + 1 \quad (\because |\vec{a} \times \vec{b}| = 1 \times 1 \times \sin 90^\circ = 1)$

$\Rightarrow 8 \cos^2 \alpha = 3$

29. (9)  $\vec{s} = 4\vec{p} + 3\vec{q} + 5\vec{r}$  ... (i)

$\vec{s} = x(-\vec{p} + \vec{q} + \vec{r}) + y(\vec{p} - \vec{q} + \vec{r}) + z(-\vec{p} - \vec{q} + \vec{r})$  ... (ii)

comparing (i) and (ii) we get

$\Rightarrow -x + y - z = 4$

$x - y - z = 3$

$x + y + z = 5$

Solving above equations, we get  $x = 4, y = \frac{9}{2}, z = \frac{-7}{2}$

$\therefore 2x + y + z = 9$

30. (4)  $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \cos \frac{\pi}{3} = \frac{1}{2}$

Given  $p\vec{a} + q\vec{b} + r\vec{c} = \vec{a} \times \vec{b} + \vec{b} \times \vec{c}$

Taking its dot product with  $\vec{a}, \vec{b}, \vec{c}$ , we get

$$\begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix} = p|\vec{a}|^2 + q(\vec{b} \cdot \vec{a}) + r(\vec{c} \cdot \vec{a})$$

$= p + \frac{1}{2}q + \frac{1}{2}r$  ... (i)

Given that  $\frac{1}{2}p + q + \frac{1}{2}r = 0$  ... (ii)

and  $\frac{1}{2}p + \frac{1}{2}q + r = \begin{bmatrix} \vec{a} & \vec{b} & \vec{c} \end{bmatrix}$  ... (iii)

From (i) and (iii),  $p = r$  Using (ii)  $q = -p$

$\therefore \frac{p^2 + 2q^2 + r^2}{q^2} = \frac{p^2 + 2p^2 + p^2}{p^2} = 4$

31. (9)  $\because \vec{r} \times \vec{b} = \vec{c} \times \vec{b} \Rightarrow \vec{r} \times \vec{b} - \vec{c} \times \vec{b} = \vec{0}$

$\Rightarrow (\vec{r} - \vec{c}) \times \vec{b} = \vec{0} \Rightarrow \vec{r} - \vec{c} \parallel \vec{b}$

Let  $\vec{r} - \vec{c} = \lambda \vec{b} \Rightarrow \vec{r} = \vec{c} + \lambda \vec{b}$

$\Rightarrow \vec{r} = \hat{i} + 2\hat{j} + 3\hat{k} - \lambda\hat{i} + \lambda\hat{j}$

$= (1 - \lambda)\hat{i} + (2 + \lambda)\hat{j} + 3\hat{k}$

$\therefore \vec{r} \cdot \vec{a} = 0 \Rightarrow -1 + \lambda - 3 = 0 \Rightarrow \lambda = 4$

$\therefore \vec{r} = -3\hat{i} + 6\hat{j} + 3\hat{k}$

Now,  $\vec{r} \cdot \vec{b} = 3 + 6 = 9$

32. (5) We have  $|\vec{a}| = 1, |\vec{b}| = 1$  and  $\vec{a} \cdot \vec{b} = 0$

Now,  $(2\vec{a} + \vec{b}) \cdot [(\vec{a} \times \vec{b}) \times (\vec{a} - 2\vec{b})]$

$= (2\vec{a} + \vec{b}) \cdot [(\vec{a} \cdot \vec{a})\vec{b} - (\vec{a} \cdot \vec{b})\vec{a} - 2(\vec{b} \cdot \vec{a})\vec{b} + 2(\vec{b} \cdot \vec{b})\vec{a}]$

$= (2\vec{a} + \vec{b}) \cdot [(\vec{a})^2 \vec{b} + 2(\vec{b})^2 \vec{a}]$

$= (2\vec{a} + \vec{b}) \cdot (2\vec{a} + \vec{b}) = 4|\vec{a}|^2 + |\vec{b}|^2 \quad (\because \vec{a} \cdot \vec{b} = 0)$

$= 4 + 1 = 5.$

33. (18) Given that  $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

$\Rightarrow \vec{a} + \vec{b} = 3\hat{i} + 3\hat{j}$

Given that projection of  $\vec{c}$  on  $\vec{a} + \vec{b} = 3\sqrt{2}$

$\Rightarrow \frac{\vec{c} \cdot (\vec{a} + \vec{b})}{|\vec{a} + \vec{b}|} = 3\sqrt{2} \Rightarrow \frac{3(2\alpha + \beta) + 3(\alpha + 2\beta)}{3\sqrt{2}} = 3\sqrt{2}$

$\Rightarrow \alpha + \beta = 2$  ..... (i)

$\vec{c} = \alpha\vec{a} + \beta\vec{b} = (2\alpha + \beta)\hat{i} + (\alpha + 2\beta)\hat{j} + (-\alpha + \beta)\hat{k}$

$\vec{c} = (\alpha + 2)\hat{i} + (4 - \alpha)\hat{j} + (2 - 2\alpha)\hat{k}$  Using eq<sup>n</sup> (i)

Now  $\vec{c}$  is in the plane of  $\vec{a}$  and  $\vec{b}$  ( $\because \vec{c} = \alpha\vec{a} + \beta\vec{b}$ )

$\therefore (\vec{a} \times \vec{b}) \cdot \vec{c} = 0$

Hence  $(\vec{c} - (\vec{a} \times \vec{b})) \cdot \vec{c} = \vec{c} \cdot \vec{c}$

$= (\alpha + 2)^2 + (4 - \alpha)^2 + (2 - 2\alpha)^2 = 6(\alpha^2 - 2\alpha + 4)$

$= 6((\alpha - 1)^2 + 3)$

which has minimum value as 18 when  $\alpha = 1$

34. Given that  $q =$  area of parallelogram with  $\vec{OA}$  and  $\vec{OC}$  as

adjacent sides  $= |\vec{OA} \times \vec{OC}| = |\vec{a} \times \vec{b}|$

and  $p =$  area of quadrilateral  $OABC$

$$\begin{aligned} &= \frac{1}{2} |\overline{OA} \times \overline{OB}| + \frac{1}{2} |\overline{OB} \times \overline{OC}| \\ &= \frac{1}{2} |\vec{a} \times (\overline{10a} + \overline{2b})| + \frac{1}{2} |(\overline{10a} + \overline{2b}) \times \vec{b}| \\ &= |\vec{a} \times \vec{b}| + 5|\vec{a} \times \vec{b}| = 6|\vec{a} \times \vec{b}| \quad \therefore p = 6q \Rightarrow k = 6 \end{aligned}$$

35. Let  $\vec{b}$  and  $\vec{c}$  be unit vector along  $\hat{i}$  and  $\hat{j}$  respectively

i.e.  $\vec{b} = \hat{i}$  and  $\vec{c} = \hat{j}$  then  $\vec{b} \times \vec{c} = \hat{k}$

Let  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow \vec{a} \cdot \vec{b} = x, \vec{a} \cdot \vec{c} = y$  and  $\vec{a} \cdot (\vec{b} \times \vec{c}) = k$ . Then,

$$(\vec{a} \cdot \vec{b})\vec{b} + (\vec{a} \cdot \vec{c})\vec{c} + \frac{\vec{a} \cdot (\vec{b} \times \vec{c})}{|\vec{b} \times \vec{c}|} (\vec{b} \times \vec{c}) = x\hat{i} + y\hat{j} + z\hat{k} = \vec{a}$$

36. Let  $\vec{a} = x\hat{i} + y\hat{j} + z\hat{k}$  be a unit vector, coplanar with  $\hat{i} + \hat{j} + 2\hat{k}$  and  $\hat{i} + 2\hat{j} + \hat{k}$ .

$$\Rightarrow \begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{vmatrix} = 0 \Rightarrow -3x + y + z = 0 \quad \dots(i)$$

also  $\vec{a}$  is perpendicular to  $\hat{i} + \hat{j} + \hat{k}$

$$\begin{aligned} &\Rightarrow (x\hat{i} + y\hat{j} + z\hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 0 \\ &\Rightarrow x + y + z = 0 \quad \dots(ii) \end{aligned}$$

Solving the above eqns., we get

$$\frac{x}{0} = \frac{y}{4} = \frac{z}{-4} \text{ or } \frac{x}{0} = \frac{y}{1} = \frac{z}{-1} = \lambda \text{ (say)}$$

$$\Rightarrow x = 0, y = \lambda, z = -\lambda \Rightarrow \vec{a} = \lambda\hat{j} - \lambda\hat{k} = 0$$

Given that  $\vec{a}$  is a unit vector, therefore

$$0 + \lambda^2 + \lambda^2 = 1 \Rightarrow \lambda^2 = \frac{1}{2} \Rightarrow \lambda = \pm \frac{1}{\sqrt{2}}$$

$\therefore$  The required vector is  $\frac{\hat{j} - \hat{k}}{\sqrt{2}}$  or  $\frac{-\hat{j} + \hat{k}}{\sqrt{2}}$

37. Given that the vectors  $\vec{a} = a\hat{i} + \hat{j} + \hat{k}$ ,  $\vec{b} = \hat{i} + b\hat{j} + \hat{k}$  and

$\vec{c} = \hat{i} + \hat{j} + c\hat{k}$  where  $a \neq b \neq c \neq 1$  are coplanar

$$\therefore [\vec{a} \vec{b} \vec{c}] = 0 \Rightarrow \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$$

Applying  $C_1 = C_1 - C_2, C_2 = C_2 - C_3$

$$\begin{vmatrix} a-1 & 0 & 1 \\ 1-b & b-1 & 1 \\ 0 & 1-c & c \end{vmatrix} = 0$$

Taking  $(1-a), (1-b), (1-c)$  common from  $R_1, R_2$  and  $R_3$  respectively, we get

$$\Rightarrow (1-a)(1-b)(1-c) \begin{vmatrix} -1 & 0 & \frac{1}{1-a} \\ 1 & -1 & \frac{1}{1-b} \\ 0 & 1 & \frac{c}{1-c} \end{vmatrix} = 0$$

Applying  $R_2 \rightarrow R_2 + R_1$

$$\Rightarrow (1-a)(1-b)(1-c) \begin{vmatrix} -1 & 0 & \frac{1}{1-a} \\ 0 & -1 & \frac{1}{1-b} + \frac{1}{1-a} \\ 0 & 1 & \frac{c}{1-c} \end{vmatrix} = 0$$

$$\Rightarrow (1-a)(1-b)(1-c)(-1) \left[ -\frac{c}{1-c} - \frac{1}{1-b} - \frac{1}{1-a} \right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c) \left[ \frac{1}{1-a} + \frac{1}{1-b} + \frac{c}{1-c} \right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c) \left[ \frac{1}{1-a} + \frac{1}{1-b} - \frac{(1-c)-1}{1-c} \right] = 0$$

$$\Rightarrow (1-a)(1-b)(1-c) \left[ \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1 \right] = 0$$

But  $a \neq b \neq c \neq 1$

$$\therefore \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} - 1 = 0 \Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

38. Position vector of  $\vec{A}$  and  $\vec{C}$  are  $\vec{A} = \hat{i} + \hat{j} + \hat{k}$  and  $\vec{C} = \hat{j} - \hat{k}$  respectively

Let  $\vec{B} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{ATQ, } \vec{A} \times \vec{B} = \vec{C} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ x & y & z \end{vmatrix} = \hat{j} - \hat{k}$$

$$\Rightarrow (z-y)\hat{i} - (z-x)\hat{j} + (y-x)\hat{k} = \hat{j} - \hat{k}$$

$$z-y=0 \Rightarrow y=z \quad \dots(i)$$

$$\Rightarrow x-z=1 \Rightarrow x=z+1 \quad \dots(ii)$$

$$\text{and } y-x=-1$$

$$\text{Also, } \vec{A} \cdot \vec{B} = 3 \Rightarrow x+y+z=3 \quad \dots(iii)$$

From eqs. (i), (ii) and (iii), we get

$$1+z+z+z=3$$

$$\Rightarrow z = 2/3 \Rightarrow y = 2/3, x = 5/3$$

$$\therefore \vec{B} = \frac{5}{3}\hat{i} + \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$$

39. Since  $\vec{A}, \vec{B}, \vec{C}$  are three non-coplanar vectors, therefore,

$$[\vec{A} \vec{B} \vec{C}] \neq 0$$

$$\therefore \frac{\vec{A} \cdot (\vec{B} \times \vec{C})}{(\vec{C} \times \vec{A}) \cdot \vec{B}} + \frac{\vec{B} \cdot (\vec{A} \times \vec{C})}{(\vec{C} \times \vec{A}) \cdot \vec{B}} = \frac{[\vec{A} \vec{B} \vec{C}]}{[\vec{C} \vec{A} \vec{B}]} + \frac{[\vec{B} \vec{A} \vec{C}]}{[\vec{C} \vec{A} \vec{B}]}$$

$$= \frac{[\vec{A} \vec{B} \vec{C}]}{[\vec{A} \vec{B} \vec{C}]} + \frac{-[\vec{A} \vec{B} \vec{C}]}{[\vec{A} \vec{B} \vec{C}]} = 0$$



40. 
$$\begin{vmatrix} a & a^2 & 1+a^3 \\ b & b^2 & 1+b^3 \\ c & c^2 & 1+c^3 \end{vmatrix} = 0 \quad \text{(Given)}$$

$$\Rightarrow \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

Apply  $C_2 \leftrightarrow C_3$  and then  $C_1 \leftrightarrow C_2$  in first determinant, we get

$$\Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} + abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$\Rightarrow (1+abc) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

But given that the vectors  $\vec{A}, \vec{B}, \vec{C}$  are non-coplanar

$$\therefore 1+abc = 0 \Rightarrow \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} \neq 0$$

$$\Rightarrow abc = -1$$

41.  $\therefore \vec{BA} = -\hat{i} - 2\hat{j} + 3\hat{k}, \vec{BC} = \hat{i} - 2\hat{j} + 3\hat{k}$

$$\therefore \text{Area of } \Delta ABC = \frac{1}{2} |\vec{BA} \times \vec{BC}|$$

$$\Delta = \frac{1}{2} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -2 & 3 \\ 1 & -2 & 3 \end{vmatrix} = \frac{1}{2} |6\hat{j} + 4\hat{k}| = |3\hat{j} + 2\hat{k}| = \sqrt{9+4} = \sqrt{13}$$

42. **False:** L.H.S.  $= (\vec{a} - \vec{b}) \cdot (\vec{b} - \vec{c}) \times (\vec{c} - \vec{a})$

$$= (\vec{a} - \vec{b}) \cdot (\vec{b} \times \vec{c} - \vec{b} \times \vec{a} - \vec{c} \times \vec{c} + \vec{c} \times \vec{a})$$

$$= (\vec{a} - \vec{b}) \cdot (\vec{b} \times \vec{c} + \vec{a} \times \vec{b} + \vec{c} \times \vec{a}) \quad [\because \vec{c} \times \vec{c} = 0]$$

$$= \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{a} \cdot (\vec{a} \times \vec{b}) + \vec{a} \cdot (\vec{c} \times \vec{a})$$

$$- \vec{b} \cdot (\vec{b} \times \vec{c}) - \vec{b} \cdot (\vec{a} \times \vec{b}) - \vec{b} \cdot (\vec{c} \times \vec{a})$$

$$= [\vec{a} \vec{b} \vec{c}] - [\vec{a} \vec{b} \vec{c}] = 0 \neq \text{R.H.S.} \quad [\because [\vec{a} \vec{a} \vec{b}] = 0]$$

$\therefore$  The given statement is false.

43. **True:**  $\because \vec{X} \cdot \vec{A} = 0 \Rightarrow$  either  $\vec{A} = 0$  or  $\vec{X} \perp \vec{A}$  ... (i)

$$\vec{X} \cdot \vec{B} = 0 \Rightarrow \text{either } \vec{B} = 0 \text{ or } \vec{X} \perp \vec{B} \quad \dots \text{(ii)}$$

$$\vec{X} \cdot \vec{C} = 0 \Rightarrow \text{either } \vec{C} = 0 \text{ or } \vec{X} \perp \vec{C} \quad \dots \text{(iii)}$$

From (i), (ii) and (iii)

$$\text{if } \vec{A} \text{ or } \vec{B} \text{ or } \vec{C} = 0 \Rightarrow [\vec{A} \vec{B} \vec{C}] = 0$$

Otherwise if  $\vec{X} \perp \vec{A}, \vec{X} \perp \vec{B}, \vec{X} \perp \vec{C}$  then  $\vec{A}, \vec{B}, \vec{C}$  are

$$\text{coplanar} \Rightarrow [\vec{A} \vec{B} \vec{C}] = 0$$

$\therefore$  Given statement is true.

44. **True:** Given that  $\vec{A} \cdot \vec{B} = \vec{A} \cdot \vec{C} = 0$

$\therefore \vec{A}$  is perpendicular to both  $\vec{B}$  and  $\vec{C}$ .

$$\Rightarrow \vec{B} \times \vec{C} = \lambda \vec{A} \text{ where } \lambda \text{ is any scalar.}$$

$$\Rightarrow |\vec{B} \times \vec{C}| = |\lambda \vec{A}| \Rightarrow \sin \pi/6 = \pm \lambda \quad (\because |\vec{B}| = |\vec{C}| = |\vec{A}| = 1)$$

( $\because \pi/6$  is the angle between  $\vec{B}$  &  $\vec{C}$ )

$$\Rightarrow \lambda = \pm \frac{1}{2} \Rightarrow \vec{B} \times \vec{C} = \pm \frac{1}{2} \vec{A} \Rightarrow \vec{A} = \pm 2(\vec{B} \times \vec{C})$$

$\therefore$  Given statement is true.

45. (b, c, d) 
$$\begin{bmatrix} 0 & -C_3 & C_2 \\ C_3 & 0 & -C_1 \\ -C_2 & C_1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 3-C_1 \\ 1-C_2 \\ -1-C_3 \end{bmatrix}$$

$$\Rightarrow -b_2 C_3 + b_3 C_2 = 3 - C_1 \quad \dots \text{(i)}$$

$$\Rightarrow C_3 - C_1 b_3 = 1 - C_2 \quad \dots \text{(ii)}$$

$$\Rightarrow -C_2 + b_2 C_1 = -1 - C_3 \quad \dots \text{(iii)}$$

Applying (i)  $\hat{i} - (2)\hat{j} + (3)\hat{k}$ , we get

$$i(b_2 c_3 - b_3 c_2) - j(c_3 - b_3 c_1) + k(c_2 - b_2 c_1)$$

$$= c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k} - 3\hat{i} - \hat{j} + \hat{k} \Rightarrow \vec{b} \times \vec{c} = \vec{c} - \vec{a}$$

$$\Rightarrow \vec{b} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} - \vec{a}) \Rightarrow 0 = \vec{b} \cdot \vec{c} - \vec{b} \cdot \vec{a}$$

$$\Rightarrow \vec{b} \cdot \vec{c} = 0 \quad [\because \vec{a} \cdot \vec{b} = 0]$$

$$\therefore \vec{c} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot (\vec{c} - \vec{a})$$

$$\Rightarrow 0 = |\vec{c}|^2 - \vec{c} \cdot \vec{a} \Rightarrow \vec{c} \cdot \vec{a} = |\vec{c}|^2$$

$$\therefore \vec{a} \cdot \vec{c} \neq 0$$

$$(\vec{b} \times \vec{c})^2 = (\vec{c} - \vec{a})^2$$

$$|\vec{b}|^2 |\vec{c}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} \quad [\because \vec{b} \perp \vec{c}]$$

$$|\vec{b}|^2 |\vec{c}|^2 = |\vec{c}|^2 + 11 - 2|\vec{c}|^2$$

$$\Rightarrow |\vec{c}|^2 = \frac{11}{|\vec{b}|^2 + 1} \Rightarrow |\vec{c}| \leq \sqrt{11}$$

$$\text{Given that } \vec{a} \cdot \vec{b} = 0 \Rightarrow b_2 - b_3 + 3 = 0 \Rightarrow b_3 - b_2 = 3$$

Also  $b_2, b_3 > 0$

$$\text{Now, } |\vec{b}|^2 = 1 + b_2^2 + b_3^2$$

$$= 1 + (b_3 - b_2)^2 + 2b_2 b_3$$

$$= 10 + 2b_2 b_3$$

$$\Rightarrow |\vec{b}|^2 > 10 \Rightarrow |\vec{b}| > \sqrt{10}$$

46. (a, b, c) 
$$\vec{OB} \times \vec{OC} = \frac{1}{2} \vec{OB} \times (\vec{OB} - \lambda \vec{OA})$$

$$= \frac{\lambda}{2} (\vec{OA} \times \vec{OB}) \quad \dots \text{(i)}$$



Since,  $\vec{OA} \cdot \vec{OB} = 0$  therefore  $\vec{OA} \perp \vec{OB}$

$$\therefore |\vec{OB} \times \vec{OC}| = \left| \frac{\lambda}{2} (\vec{OA} \times \vec{OB}) \right| \quad [\text{from (i)}]$$

$$= \frac{|\lambda|}{2} |\vec{OA}| |\vec{OB}| \sin \frac{\pi}{2}$$

$$\Rightarrow \frac{9}{2} = \frac{|\lambda|}{2} \times 3 \times 3 \Rightarrow \lambda = 1 \text{ (given } \lambda > 0)$$

$$\text{So, } \vec{OC} = \frac{\vec{OB} - \vec{OA}}{2} = \frac{1}{2}(-\hat{i} - 4\hat{j} + \hat{k})$$

(a) Projection  $\vec{OC}$  on  $\vec{OA}$

$$= \frac{\vec{OC} \cdot \vec{OA}}{|\vec{OA}|} = \frac{\frac{1}{2}(-2-8+1)}{3} = -\frac{3}{2}$$

(b) Area of  $\Delta OAB = \frac{1}{2} |\vec{OA} \times \vec{OB}| = \frac{9}{2}$

(c) Area of  $\Delta ABC$

$$= \frac{1}{2} |\vec{AB} \times \vec{AC}| = \left| \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 1 \\ -\frac{5}{2} & -4 & -\frac{1}{2} \end{vmatrix} \right|$$

$$= \frac{1}{2} |6\hat{i} - 3\hat{j} - 6\hat{k}| = \frac{9}{2}$$

(d) Let acute angle between diagonals is  $\theta$  then

$$\cos \theta = \frac{(\vec{OA} + \vec{OC}) \cdot (\vec{OA} - \vec{OC})}{|\vec{OA} + \vec{OC}| |\vec{OA} - \vec{OC}|} = \frac{18}{3\sqrt{2}\sqrt{90}}$$

$$\therefore \theta \neq \frac{\pi}{3}$$

47. (b, c, d)  $\begin{bmatrix} 0 & -C_3 & C_2 \\ C_3 & 0 & -C_1 \\ -C_2 & C_1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 3 - C_1 \\ 1 - C_2 \\ -1 - C_3 \end{bmatrix}$

$$\Rightarrow -b_2 C_3 + b_3 C_2 = 3 - C_1 \quad \dots\dots (i)$$

$$\Rightarrow C_3 - C_1 b_3 = 1 - C_2 \quad \dots\dots (ii)$$

$$\Rightarrow -C_2 + b_2 C_1 = -1 - C_3 \quad \dots\dots (iii)$$

Applying (i)  $\hat{i} - (2)\hat{j} + (3)\hat{k}$ , we get

$$i(b_2 c_3 - b_3 c_2) - j(c_3 - b_3 c_1) + k(c_2 - b_2 c_1)$$

$$= c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k} - 3\hat{i} - \hat{j} + \hat{k} \Rightarrow \vec{b} \times \vec{c} = \vec{c} - \vec{a}$$

$$\Rightarrow \vec{b} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} - \vec{a}) \Rightarrow 0 = \vec{b} \cdot \vec{c} - \vec{b} \cdot \vec{a}$$

$$\Rightarrow \vec{b} \cdot \vec{c} = 0 \quad [\because \vec{a} \cdot \vec{b} = 0]$$

$$\therefore \vec{c} \cdot (\vec{b} \times \vec{c}) = \vec{c} \cdot (\vec{c} - \vec{a}) \Rightarrow 0 = |\vec{c}|^2 - \vec{c} \cdot \vec{a} \Rightarrow \vec{c} \cdot \vec{a} = |\vec{c}|^2$$

$$\therefore \vec{a} \cdot \vec{c} \neq 0$$

$$(\vec{b} \times \vec{c})^2 = (\vec{c} - \vec{a})^2$$

$$|\vec{b}|^2 |\vec{c}|^2 = |\vec{c}|^2 + |\vec{a}|^2 - 2\vec{c} \cdot \vec{a} \quad [\because \vec{b} \perp \vec{c}]$$

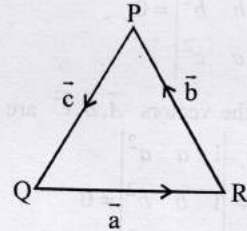
$$|\vec{b}|^2 |\vec{c}|^2 = |\vec{c}|^2 + 11 - 2|\vec{c}|^2$$

$$\Rightarrow |\vec{c}|^2 = \frac{11}{|\vec{b}|^2 + 1} \Rightarrow |\vec{c}| \leq \sqrt{11}$$

Given that  $\vec{a} \cdot \vec{b} = 0 \Rightarrow b_2 - b_3 + 3 = 0 \Rightarrow b_3 - b_2 = 3$   
Also  $b_2, b_3 > 0$

$$\text{Now, } |\vec{b}|^2 = 1 + b_2^2 + b_3^2 = 1 + (b_3 - b_2)^2 + 2b_2 b_3 = 10 + 2b_2 b_3$$

48.  $\Rightarrow |\vec{b}|^2 > 10 \Rightarrow |\vec{b}| > \sqrt{10}$   
(a, c, d)  
From given information  $\vec{a} + \vec{b} + \vec{c} = \vec{0}$



$$\Rightarrow |\vec{b} + \vec{c}|^2 = |\vec{a}|^2 \Rightarrow |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} = |\vec{a}|^2$$

$$\Rightarrow 48 + |\vec{c}|^2 + 48 = 144 \Rightarrow |\vec{c}|^2 = 48 \Rightarrow |\vec{c}| = 4\sqrt{3}$$

$$\therefore \frac{|\vec{c}|^2}{2} - |\vec{a}|^2 = \frac{48}{2} - 12 = 12$$

$\therefore$  (a) is correct

$$\frac{|\vec{c}|^2}{2} + |\vec{a}|^2 = 24 \neq 30$$

$\therefore$  (b) is not correct

$$\text{Also } |\vec{b}| = |\vec{c}| \Rightarrow \angle Q = \angle R$$

$$\text{and } \cos(180 - P) = \frac{\vec{b} \cdot \vec{c}}{|\vec{b}| |\vec{c}|} = \frac{1}{2}$$

$$\Rightarrow \angle P = 120^\circ \therefore \angle Q = \angle R = 30^\circ$$

$$\text{And } \vec{a} \cdot \vec{b} = 12 \times 4\sqrt{3} \times \cos 150 = -72$$

$\therefore$  (d) is correct

$$\text{Now, } \vec{a} \times (\vec{a} + \vec{b} + \vec{c}) = \vec{0} \Rightarrow \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$$

$$\therefore |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2|\vec{a} \times \vec{b}| = 2 \times 12 \times 4\sqrt{3} \times \sin 150 = 48\sqrt{3}$$

$\therefore$  (c) is correct.

49. (a, b, c) Given that  $|\vec{x}| = |\vec{y}| = |\vec{z}| = \sqrt{2}$

and angle between each pair is  $\frac{\pi}{3}$ .

Since  $\vec{a}$  is perpendicular to both  $\vec{x}$  and  $\vec{y} \times \vec{z}$ .

$$\begin{aligned} \therefore \vec{a} &= \lambda \left[ \vec{x} \times \left( \vec{y} \times \vec{z} \right) \right] = \lambda \left[ \left( \vec{x} \cdot \vec{z} \right) \vec{y} - \left( \vec{x} \cdot \vec{y} \right) \vec{z} \right] \\ &= \lambda \left[ \left( \sqrt{2} \cdot \sqrt{2} \cos \frac{\pi}{3} \right) \vec{y} - \left( \sqrt{2} \cdot \sqrt{2} \cos \frac{\pi}{3} \right) \vec{z} \right] \\ &= \lambda \left( \vec{y} - \vec{z} \right) \end{aligned}$$

Since  $\vec{b}$  is perpendicular to both  $\vec{y}$  and  $\vec{z} \times \vec{x}$ .

$$\begin{aligned} \vec{b} &= \mu \left[ \vec{y} \times \left( \vec{z} \times \vec{x} \right) \right] = \mu \left[ \left( \vec{y} \cdot \vec{x} \right) \vec{z} - \left( \vec{y} \cdot \vec{z} \right) \vec{x} \right] \\ &= \mu \left[ \left( \sqrt{2} \cdot \sqrt{2} \cos \frac{\pi}{3} \right) \vec{z} - \left( \sqrt{2} \cdot \sqrt{2} \cos \frac{\pi}{3} \right) \vec{x} \right] \\ &= \mu \left( \vec{z} - \vec{x} \right) \end{aligned}$$

$$\text{Now, } \vec{b} \cdot \vec{z} = \mu \left[ \vec{z} \cdot \vec{z} - \vec{x} \cdot \vec{z} \right] = \mu (2 - 1) = \mu$$

$$\therefore \vec{b} = \left( \vec{z} - \vec{x} \right) \left( \vec{z} - \vec{x} \right)$$

$\therefore$  (a) is correct

$$\text{Also } \vec{a} \cdot \vec{y} = \lambda \left( \vec{y} \cdot \vec{y} - \vec{z} \cdot \vec{y} \right) = \lambda (2 - 1) = \lambda$$

$$\therefore \vec{a} = \left( \vec{y} - \vec{z} \right) \left( \vec{y} - \vec{z} \right)$$

$\therefore$  (b) is also correct

$$\vec{a} \cdot \vec{b} = \lambda \mu \left( \vec{y} \cdot \vec{z} - \vec{y} \cdot \vec{x} - \vec{z} \cdot \vec{z} + \vec{z} \cdot \vec{x} \right)$$

$$= \lambda \mu (1 - 1 - 2 + 1) = -\lambda \mu = - \left( \vec{a} \cdot \vec{y} \right) \left( \vec{b} \cdot \vec{z} \right)$$

$\therefore$  (c) is correct.

$$- \left( \vec{a} \cdot \vec{y} \right) \left( \vec{z} - \vec{y} \right) = \lambda \left( \vec{z} - \vec{y} \right) = -\vec{a}$$

(d) is not correct.

50. (a,d) Let  $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$ ,  $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ ,  $\vec{c} = \hat{i} + \hat{j} + \hat{k}$

$\therefore$  Vector coplanar with  $\vec{a}$  and  $\vec{b}$  is

$$\Rightarrow \vec{r} = \vec{a} + \lambda \vec{b}$$

$$\Rightarrow \vec{r} = (1 + \lambda) \hat{i} + (1 + 2\lambda) \hat{j} + (2 + \lambda) \hat{k}$$

$$\therefore \vec{r} \perp \vec{c} \Rightarrow \vec{r} \cdot \vec{c} = 0$$

$$\Rightarrow 1 + \lambda + 1 + 2\lambda + 2 + \lambda = 0 \Rightarrow \lambda + \mu = 0 \Rightarrow \lambda = -\mu$$

$$\Rightarrow 4\lambda + 4 = 0$$

$$\Rightarrow \lambda = -1$$

Any scalar multiple of  $\vec{r}$  is also solution.

$\therefore$  a and d are the correct options.

51. (a, c) Let  $\theta$  be the angle between  $\vec{a}$  and  $\vec{b}$

$$\vec{v} = \vec{a} \times \vec{b} = |\vec{a}| |\vec{b}| \sin \theta \hat{n} = \sin \theta \hat{n}$$

[ $\therefore \vec{a}$  and  $\vec{b}$  are unit vectors.]

$$\therefore |\vec{v}| = \sin \theta \quad \dots(i)$$

$$\text{Now, } \vec{u} = \vec{a} - (\vec{a} \cdot \vec{b}) \vec{b} = \vec{a} - \vec{b} \cos \theta$$

(where  $\vec{a} \cdot \vec{b} = \cos \theta$ )

$$\therefore |\vec{u}|^2 = |\vec{a} - \vec{b} \cos \theta|^2 = 1 + \cos^2 \theta - 2 \cos \theta \cdot \cos \theta$$

$$= 1 - \cos^2 \theta = \sin^2 \theta = |\vec{v}|^2 \Rightarrow |\vec{u}| = |\vec{v}| \text{ (from (i))}$$

$$\text{Also, } \vec{u} \cdot \vec{b} = \vec{a} \cdot \vec{b} - (\vec{a} \cdot \vec{b}) (\vec{b} \cdot \vec{b}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{b} = 0,$$

$$\therefore |\vec{u} \cdot \vec{b}| = 0$$

So,  $|\vec{v}| = |\vec{u}| + |\vec{u} \cdot \vec{b}|$  is also correct

Similarly,  $|\vec{u}| + |\vec{u} \cdot \vec{a}| \neq |\vec{v}|$

and  $|\vec{u}| + |\vec{u}| |\vec{a} + \vec{b}| \neq |\vec{v}|$

52. (a, c) We know that dot product of two vectors gives a scalar quantity.

53. (c) We know that  $[\vec{u} \vec{v} \vec{w}] = [\vec{v} \vec{w} \vec{u}] = [\vec{w} \vec{u} \vec{v}]$

$$\text{but } [\vec{v} \vec{u} \vec{w}] = -[\vec{u} \vec{v} \vec{w}]$$

54. (b) Vector perpendicular to vector  $\vec{a}$  and  $\vec{b}$  is given by

$$\hat{n} = \lambda (\vec{a} \times \vec{b})$$

$$\therefore \hat{n} = \frac{\lambda (\vec{a} \times \vec{b})}{|\lambda| |\vec{a} \times \vec{b}|} = \pm \frac{(\vec{a} \times \vec{b})}{|\vec{a} \times \vec{b}|}$$

$\therefore$  We have two possible values of  $\hat{n}$

55. (c) Since 'c' is unit vector perpendicular to both the vectors  $\vec{a}$  and  $\vec{b}$ . So,  $\vec{c} \parallel \vec{a} \times \vec{b}$

$$\text{Then } \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}^2 = [\vec{a} \vec{b} \vec{c}]^2 = (\vec{a} \times \vec{b} \cdot \vec{c})^2$$

$$= (|\vec{a} \times \vec{b}| \cdot 1 \cos 0^\circ)^2$$

$$= (|\vec{a} \times \vec{b}|)^2 = \left( |\vec{a}| |\vec{b}| \cdot \sin \frac{\pi}{6} \right)^2$$

[ $\therefore$  angle between  $\vec{a}$  and  $\vec{b}$  is  $\frac{\pi}{6}$ ]

$$= \left( \frac{1}{2} \sqrt{a_1^2 + a_2^2 + a_3^2} \sqrt{b_1^2 + b_2^2 + b_3^2} \right)^2$$

$$= \frac{1}{4} (a_1^2 + a_2^2 + a_3^2) (b_1^2 + b_2^2 + b_3^2)$$

56. (a)  $P \rightarrow 4: y = \cos(3 \cos^{-1} x)$

$$y = \cos \left[ \cos^{-1} (4x^3 - 3x) \right]$$

$$y = 4x^3 - 3x$$

$$\Rightarrow \frac{dy}{dx} = 12x^2 - 3 \text{ and } \frac{d^2y}{dx^2} = 24x$$

$$\begin{aligned} \therefore \frac{1}{y} \left\{ (x^2 - 1) \frac{d^2 y}{dx^2} + x \frac{dy}{dx} \right\} \\ = \frac{1}{4x^3 - 3x} \left\{ (x^2 - 1) 24x + x(12x^2 - 3) \right\} \\ = \frac{1}{4x^3 - 3x} \{ 36x^3 - 27x \} = \frac{9(4x^3 - 3x)}{4x^3 - 3x} = 9 \end{aligned}$$

Q → 3:  $\therefore \vec{a}_1, \vec{a}_2, \dots, \vec{a}_n$  are position vectors of vertices  $A_1, A_2, A_3, \dots, A_n$  of a regular polygon of  $n$  sides with its centre at origin.

$$\therefore |\vec{a}_1| = |\vec{a}_2| = \dots = |\vec{a}_n| = \lambda$$

$$\text{Now, } \vec{a}_k \times \vec{a}_{k+1} = \lambda^2 \sin \frac{2\pi}{n} \hat{n}$$

$$\text{and } \vec{a}_k \cdot \vec{a}_{k+1} = \lambda^2 \cos \frac{2\pi}{n}$$

$$\therefore \left| \sum_{k=1}^{n-1} (\vec{a}_k \times \vec{a}_{k+1}) \right| = \left| \sum_{k=1}^{n-1} (\vec{a}_k \cdot \vec{a}_{k+1}) \right|$$

$$\Rightarrow (n-1) \lambda^2 \sin \frac{2\pi}{n} = (n-1) \lambda^2 \cos \frac{2\pi}{n}$$

$$\Rightarrow \tan \frac{2\pi}{n} = 1 \Rightarrow \frac{2\pi}{n} = \frac{\pi}{4} \Rightarrow n = 8$$

R → 2: Normal from  $P(h, 1)$  on  $\frac{x^2}{6} + \frac{y^2}{3} = 1$  is

$$\frac{x-h}{h/6} = \frac{y-1}{1/3}$$

$$\Rightarrow 2(x-h) = h(y-1) \Rightarrow 2x - hy - h = 0$$

$$\text{Slope of Normal} = \frac{2}{h}$$

It is perpendicular to  $x + y = 8$

$$\therefore \frac{2}{h} \times -1 = -1 \Rightarrow h = 2$$

$$S \rightarrow 1: \tan^{-1} \left( \frac{1}{2x+1} \right) + \tan^{-1} \left( \frac{1}{4x+1} \right) = \tan^{-1} \left( \frac{2}{x^2} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{\frac{1}{2x+1} + \frac{1}{4x+1}}{1 - \frac{1}{2x+1} \cdot \frac{1}{4x+1}} \right) = \tan^{-1} \left( \frac{2}{x^2} \right)$$

$$\Rightarrow \tan^{-1} \left( \frac{6x+2}{8x^2+6x} \right) = \tan^{-1} \left( \frac{2}{x^2} \right)$$

$$\Rightarrow \frac{3x+1}{4x^2+3x} = \frac{2}{x^2}$$

$$\Rightarrow 3x^2 - 7x - 6 = 0 \Rightarrow x = 3 \text{ or } -\frac{2}{3}$$

Since  $x > 0$

$\therefore$  Only one +ve solution is there

Hence (a) is the correct option.

57. (c) (P) Given that  $[\vec{a} \vec{b} \vec{c}] = 2$

$$\therefore [2(\vec{a} \times \vec{b}) \ 3(\vec{b} \times \vec{c}) \ \vec{c} \times \vec{a}]$$

$$= 6[\vec{a} \times \vec{b} \ \vec{b} \times \vec{c} \ \vec{c} \times \vec{a}]$$

$$= 6[\vec{a} \ \vec{b} \ \vec{c}]^2 = 6 \times 4 = 24 \therefore (P) \rightarrow (3)$$

(Q) Given that  $[\vec{a} \ \vec{b} \ \vec{c}] = 5$

$$\therefore [3(\vec{a} + \vec{b}) \ \vec{b} + \vec{c} \ 2(\vec{c} + \vec{a})]$$

$$= 6[\vec{a} + \vec{b} \ \vec{b} + \vec{c} \ \vec{c} + \vec{a}]$$

$$= 6 \times 2[\vec{a} \ \vec{b} \ \vec{c}] = 6 \times 2 \times 5 = 60$$

$$\therefore (Q) \rightarrow (4)$$

(R) Given that  $\frac{1}{2}|\vec{a} \times \vec{b}| = 20 \Rightarrow |\vec{a} \times \vec{b}| = 40$

$$\therefore \frac{1}{2} |(2\vec{a} + 3\vec{b}) \times (\vec{a} - \vec{b})| = \frac{1}{2} |-2\vec{a} \times \vec{b} + 3\vec{b} \times \vec{a}|$$

$$= \frac{1}{2} \times 5 |\vec{a} \times \vec{b}| = \frac{5}{2} \times 40 = 100$$

$$\therefore (R) \rightarrow (1)$$

(S) Given that  $|\vec{a} \times \vec{b}| = 30$

$$\therefore |(\vec{a} + \vec{b}) \times \vec{a}| = |(\vec{b} \times \vec{a})| = 30$$

$$\therefore (S) \rightarrow (2)$$

58. **A** → **q, s**; **B** → **p, r, s, t**; **C** → **t**; **D** → **r**

(A) The given equation is

$$2 \sin^2 \theta + \sin^2 2\theta = 2$$

$$\Rightarrow 2 \sin^2 \theta + 4 \sin^2 \theta \cos^2 \theta - 2 = 0$$

$$\Rightarrow \sin^2 \theta + 2 \sin^2 \theta (1 - \sin^2 \theta) - 1 = 0$$

$$\Rightarrow 2 \sin^4 \theta - 3 \sin^2 \theta + 1 = 0$$

$$\Rightarrow 2 \sin^4 \theta - 2 \sin^2 \theta - \sin^2 \theta + 1 = 0$$

$$\Rightarrow 2 \sin^2 \theta (\sin^2 \theta - 1) - 1(\sin^2 \theta - 1) = 0$$

$$\Rightarrow (\sin^2 \theta - 1)(2 \sin^2 \theta - 1) = 0$$

$$\Rightarrow \sin^2 \theta = 1 \text{ or } \sin^2 \theta = \frac{1}{2}$$

$$\Rightarrow \sin \theta = \pm 1 \text{ or } \sin \theta = \pm \frac{1}{\sqrt{2}}$$

$$\Rightarrow \theta = n\pi \pm \frac{\pi}{2} \text{ or } n\pi \pm \frac{\pi}{4} \Rightarrow \theta = \frac{\pi}{2} \text{ or } \frac{\pi}{4}$$

(B) We know that  $[x]$  is discontinuous at all integral values,

therefore  $\left[ \frac{6x}{\pi} \right]$  is discontinuous at  $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$  and

$\pi$ . Also  $\cos \left[ \frac{3x}{\pi} \right] \neq 0$  for any of these values of  $x$ .

$\therefore \left[ \frac{6x}{\pi} \right] \cos \left[ \frac{3x}{\pi} \right]$  is discontinuous at  $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$  and  $\pi$ .

(C) Let  $\vec{a} = \hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j}$  and  $\vec{c} = \hat{i} + \hat{j} + \pi\hat{k}$

$\therefore$  The volume parallelepiped =  $[\vec{a} \vec{b} \vec{c}]$

$$= \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 1 & 1 & \pi \end{vmatrix} = \pi$$

(D)  $\because \vec{a} + \vec{b} = -\sqrt{3}\vec{c} \Rightarrow |\vec{a} + \vec{b}|^2 = 3|\vec{c}|^2$

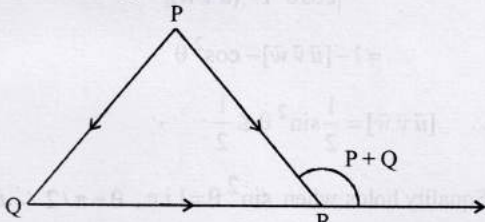
$$\Rightarrow (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b}) = 3$$

$$\Rightarrow |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a} \cdot \vec{b} = 3 \Rightarrow 1 + 1 + 2\cos\theta = 3$$

(where  $\theta$  is the angle between  $\vec{a}$  and  $\vec{b}$ )

$$\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

59. (a)  $\vec{OX}, \vec{OY}, \vec{OZ}$  are unit vectors in the directions of sides  $\overline{QR}, \overline{RP}$  and  $\overline{PQ}$  respectively,



$$\therefore \vec{OX} = \frac{\overline{QR}}{|\overline{QR}|}, \vec{OY} = \frac{\overline{RP}}{|\overline{RP}|}, \vec{OZ} = \frac{\overline{PQ}}{|\overline{PQ}|}$$

$$\therefore |\vec{OX} \times \vec{OY}| = \frac{|\overline{QR} \times \overline{RP}|}{|\overline{QR}||\overline{RP}|} = \frac{|\overline{QR}||\overline{RP}|\sin(P+Q)}{|\overline{QR}||\overline{RP}|}$$

$$= \sin(P+Q)$$

60. (b)  $\cos(P+Q) + \cos(Q+R) + \cos(R+P)$   
 $= \cos(180-R) + \cos(180-P) + \cos(180-Q)$   
 $= -[\cos P + \cos Q + \cos R]$

To minimize the expression we need to maximize  $\cos P + \cos Q + \cos R$ .

We know that  $\cos P + \cos Q + \cos R$  will be maximum when  $\cos P = \cos Q = \cos R$

$$\Rightarrow P = Q = R = \frac{\pi}{3}$$

$$\therefore \text{Minimum value} = -3 \cos \frac{\pi}{3} = \frac{-3}{2}$$

61. (c)  $\vec{PQ} \times (\vec{RS} + \vec{ST}) = \vec{PQ} \times \vec{RT}$  (using triangle law)

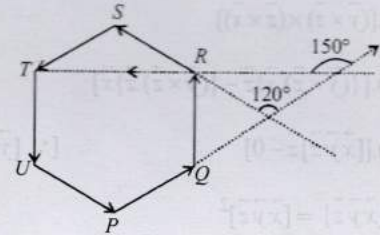
$$= |\vec{PQ}| \times |\vec{RT}| \sin 150^\circ \hat{n} \neq 0$$

Statement-1 is true.

$$\text{Also, } \vec{PQ} \times \vec{RS} = |\vec{PQ}| \times |\vec{RS}| \sin 120^\circ \times \hat{n}_1 \neq 0$$

$$\text{And } \vec{PQ} \times \vec{ST} = |\vec{PQ}| \times |\vec{ST}| \sin 180^\circ \times \hat{n}_2 = 0$$

$\therefore$  Statement-2 is false.



62. Given that

$$\vec{a} \times \vec{c} = \vec{b} \times \vec{d} \quad \dots(i)$$

$$\vec{a} \times \vec{b} = \vec{c} \times \vec{d} \quad \dots(ii)$$

Subtracting eq<sup>n</sup> (ii) from (i) we get

$$\vec{a} \times (\vec{c} - \vec{b}) = (\vec{b} - \vec{c}) \times \vec{d} \Rightarrow \vec{a} \times (\vec{c} - \vec{b}) = \vec{d} \times (\vec{c} - \vec{b})$$

$$\Rightarrow \vec{a} \times (\vec{c} - \vec{b}) - \vec{d} \times (\vec{c} - \vec{b}) = \vec{0}$$

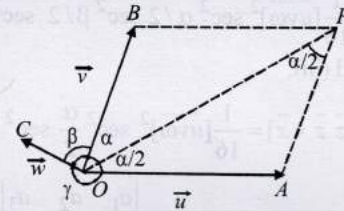
$$\Rightarrow (\vec{a} - \vec{d}) \times (\vec{c} - \vec{b}) = \vec{0} \Rightarrow (\vec{a} - \vec{d}) \parallel (\vec{c} - \vec{b})$$

$$[\because \vec{a} - \vec{d} \neq \vec{0}, \vec{c} - \vec{b} \neq \vec{0} \text{ as } \vec{a} \neq \vec{b} \neq \vec{c} \neq \vec{d}]$$

$\Rightarrow$  Angle between  $\vec{a} - \vec{d}$  and  $\vec{c} - \vec{b}$  is either 0 or 180°.

$\Rightarrow (\vec{a} - \vec{d}) \cdot (\vec{c} - \vec{b}) = |\vec{a} - \vec{d}| |\vec{c} - \vec{b}| \cos 0$  [or  $\cos 180^\circ$ ]  $\neq 0$  as  $a, b, c, d$  all are different. Hence Proved.

63.  $\vec{u}, \vec{v}$  and  $\vec{w}$  represent in figure as  $\vec{OA}, \vec{OB}$  and  $\vec{OC}$  resp. Let  $P$  be a pt. on angle bisector of  $\angle AOB$  such that  $OAPB$  is a parallelogram.



$$\therefore \angle POA = \angle BOP = \alpha/2$$

$$\therefore \angle APO = \angle BOP = \alpha/2 \quad (\text{alt. int. } \angle's)$$

$$\therefore \text{In } \triangle OAP, OA = AP$$

$$\therefore \vec{OP} = \vec{OA} + \vec{AP} = \vec{u} + \vec{v}$$

$\therefore$  A unit vector in the direction of

$$\vec{OP} = \frac{\vec{u} + \vec{v}}{|\vec{u} + \vec{v}|} \quad \text{i.e. } \vec{x} = \frac{\vec{u} + \vec{v}}{|\vec{u} + \vec{v}|}$$

$$\text{But } |\vec{u} + \vec{v}|^2 = (\vec{u} + \vec{v}) \cdot (\vec{u} + \vec{v}) = 1 + 1 + 2\vec{u} \cdot \vec{v}$$

$$[\because |\vec{u}| = |\vec{v}| = 1]$$

$$= 2 + 2\cos\alpha = 4\cos^2 \alpha/2$$

$$\therefore |\vec{u} + \vec{v}| = 2\cos \alpha/2 \Rightarrow \vec{x} = \frac{1}{2}(\sec \alpha/2)(\vec{u} + \vec{v})$$

$$\text{Similarly, } \vec{y} = \frac{1}{2}\sec \frac{\beta}{2}(\vec{v} + \vec{w}) \quad \text{and} \quad \vec{z} = \frac{1}{2}\sec \frac{\gamma}{2}(\vec{w} + \vec{u})$$

$$\text{Now } [\vec{x} \times \vec{y} \cdot \vec{z}]$$

$$\begin{aligned}
 &= (\vec{x} \times \vec{y}) \cdot [(\vec{y} \times \vec{z}) \times (\vec{z} \times \vec{x})] \\
 &= (\vec{x} \times \vec{y}) \cdot [(\vec{y} \times \vec{z}) \cdot \vec{x} - (\vec{y} \times \vec{z}) \cdot \vec{x}] \\
 &= (\vec{x} \times \vec{y}) \cdot [(\vec{y} \times \vec{z}) \cdot \vec{x} - 0] \quad [\because [\vec{y} \times \vec{z}] \cdot \vec{x} = 0] \\
 &= (\vec{x} \times \vec{y}) \cdot [(\vec{y} \times \vec{z}) \cdot \vec{x}] = (\vec{x} \times \vec{y}) \cdot \vec{x} \cdot (\vec{y} \times \vec{z}) \cdot \vec{x} \dots (i)
 \end{aligned}$$

Also  $[\vec{x} \vec{y} \vec{z}] = \frac{1}{2} \left( \sec \frac{\alpha}{2} \right) (\vec{u} + \vec{v}) \frac{1}{2} \sec \beta / 2 (\vec{v} + \vec{w}) \frac{1}{2} \sec \gamma / 2 (\vec{w} + \vec{u})$

$$= \frac{1}{8} \sec \alpha / 2 \sec \beta / 2 \sec \gamma / 2 [(\vec{u} + \vec{v}) \cdot (\vec{v} + \vec{w}) \cdot (\vec{w} + \vec{u})]$$

$$\begin{aligned}
 &= \frac{1}{8} \sec \alpha / 2 \sec \beta / 2 \sec \gamma / 2 [(\vec{u} + \vec{v}) \cdot \{(\vec{v} + \vec{w}) \times (\vec{w} + \vec{u})\}] \\
 &= \frac{1}{8} \sec \alpha / 2 \sec \beta / 2 \sec \gamma / 2 [(\vec{u} + \vec{v}) \cdot (\vec{v} \times \vec{w} + \vec{v} \times \vec{u} + \vec{w} \times \vec{u} + \vec{w} \times \vec{v})] \\
 &= \frac{1}{8} \sec \alpha / 2 \sec \beta / 2 \sec \gamma / 2 [\vec{u} \cdot (\vec{v} \times \vec{w}) + \vec{v} \cdot (\vec{w} \times \vec{u})]
 \end{aligned}$$

( $\because [\vec{abc}] = 0$  when ever any two vectors are same)  
 $= 2[\vec{uvw}]$

$$[\vec{x} \vec{y} \vec{z}] = \frac{1}{4} (\sec \alpha / 2 \sec \beta / 2 \sec \gamma / 2) [\vec{uvw}] \quad \text{(from (ii))}$$

$$\therefore [\vec{uvw}]^2 = \frac{1}{16} [\vec{uvw}]^2 \sec^2 \alpha / 2 \sec^2 \beta / 2 \sec^2 \gamma / 2 \dots (iii)$$

From (i) and (iii),

$$[\vec{x} \vec{y} \vec{z}]^2 = \frac{1}{16} [\vec{uvw}]^2 \sec^2 \frac{\alpha}{2} \cdot \sec^2 \frac{\beta}{2} \sec^2 \frac{\gamma}{2}$$

64. We know that,  $V = [\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$$\begin{aligned}
 &= a_1 (b_2 c_3 - b_3 c_2) - a_2 (b_1 c_3 - c_1 b_3) + a_3 (b_1 c_2 - c_1 b_2) \\
 &= (a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2) - (a_1 b_3 c_2 + a_2 b_1 c_3 + a_3 b_2 c_1) \dots (i)
 \end{aligned}$$

Now we know that  $AM \geq GM$

$$\therefore \frac{(a_1 + b_1 + c_1) + (a_2 + b_2 + c_2) + (a_3 + b_3 + c_3)}{3}$$

$$\geq [(a_1 + b_1 + c_1)(a_2 + b_2 + c_2)(a_3 + b_3 + c_3)]^{1/3}$$

$$\Rightarrow \frac{3L}{3} \geq [(a_1 + b_1 + c_1)(a_2 + b_2 + c_2)(a_3 + b_3 + c_3)]^{1/3}$$

$$\Rightarrow L^3 \geq (a_1 + b_1 + c_1)(a_2 + b_2 + c_2)(a_3 + b_3 + c_3)$$

$$\Rightarrow L^3 \geq a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2 + 24 \text{ more such terms}$$

$$L^3 \geq a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2$$

$$[\because a_r, b_r, c_r \geq 0 \text{ for } r = 1, 2, 3]$$

$$L^3 \geq (a_1 b_2 c_3 + a_2 b_3 c_1 + a_3 b_1 c_2) - (a_1 b_3 c_2 + a_2 b_1 c_3 + a_3 b_2 c_1)$$

[By properties of inequality  
(from (i)) Hence Proved.]

$$L^3 \geq V$$

65. Since,  $\vec{w} + (\vec{w} \times \vec{u}) = \vec{v}$

$$\Rightarrow \vec{w} = \vec{v} - (\vec{w} \times \vec{u})$$

$$[\vec{u} \vec{v} \vec{w}] = (\vec{u} \times \vec{v}) \cdot (\vec{v} - \vec{w} \times \vec{u}) = (\vec{u} \times \vec{v}) \cdot (\vec{u} \times \vec{w})$$

$$= \begin{vmatrix} \vec{u} \cdot \vec{u} & \vec{u} \cdot \vec{w} \\ \vec{v} \cdot \vec{u} & \vec{v} \cdot \vec{w} \end{vmatrix}$$

Now,  $\vec{u} \cdot \vec{u} = 1$ . Let  $\theta$  is angle between  $\vec{u}$  and  $\vec{v}$ .

$$\therefore \vec{u} \cdot \vec{w} = \vec{u} \cdot (\vec{v} - \vec{w} \times \vec{u}) = \vec{u} \cdot \vec{v} - [\vec{u} \vec{w} \vec{u}] = \vec{u} \cdot \vec{v} = \cos \theta$$

$$\vec{v} \cdot \vec{w} = \vec{v} \cdot (\vec{v} - \vec{w} \times \vec{u}) = 1 - [\vec{v} \vec{w} \vec{u}] = 1 - [\vec{u} \vec{v} \vec{w}]$$

$$\begin{aligned}
 \therefore [\vec{u} \vec{v} \vec{w}] &= \begin{vmatrix} 1 & \cos \theta \\ \cos \theta & 1 - [\vec{u} \vec{v} \vec{w}] \end{vmatrix} \\
 &= 1 - [\vec{u} \vec{v} \vec{w}] - \cos^2 \theta
 \end{aligned}$$

$$\therefore [\vec{u} \vec{v} \vec{w}] = \frac{1}{2} \sin^2 \theta \leq \frac{1}{2}$$

Equality holds when  $\sin^2 \theta = 1$  i.e.,  $\theta = \pi/2 \therefore \vec{u} \perp \vec{v}$ .

66. (a) Since,  $\vec{u} \cdot \vec{v} = |\vec{u}| |\vec{v}| \cos \theta$  and  $\vec{u} \times \vec{v} = |\vec{u}| |\vec{v}| \sin \theta \hat{n}$

Let  $\theta$  be the angle between  $\vec{u}$  and  $\vec{v}$  and  $\hat{n}$  is a unit vector perpendicular to both  $\vec{u}, \vec{v}$ .

$$\begin{aligned}
 \text{L.H.S} &= |\vec{u} \cdot \vec{v}|^2 + |\vec{u} \times \vec{v}|^2 \\
 &= |\vec{u}|^2 |\vec{v}|^2 \cos^2 \theta + |\vec{u}|^2 |\vec{v}|^2 \sin^2 \theta |\hat{n}|^2 \\
 &= |\vec{u}|^2 |\vec{v}|^2 (\cos^2 \theta + \sin^2 \theta) \quad (\because |\hat{n}| = 1) \\
 &= |\vec{u}|^2 |\vec{v}|^2 = \text{R.H.S}
 \end{aligned}$$

(b)  $|\vec{u} + \vec{v} + (\vec{u} \times \vec{v})|^2 = |\vec{u} + \vec{v}|^2 + |\vec{u} \times \vec{v}|^2 + 2(\vec{u} + \vec{v}) \cdot (\vec{u} \times \vec{v})$

$$= |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{u} \times \vec{v}|^2 + 0$$

$$\therefore \text{R.H.S} = |\vec{u} + \vec{v} + \vec{u} \times \vec{v}|^2 + |1 - \vec{u} \cdot \vec{v}|^2$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + 2\vec{u} \cdot \vec{v} + |\vec{u} \times \vec{v}|^2 + 1 - 2\vec{u} \cdot \vec{v} + |\vec{u} - \vec{v}|^2$$

$$= |\vec{u}|^2 + |\vec{v}|^2 + 1 + |\vec{u}|^2 |\vec{v}|^2 = (1 + |\vec{u}|^2)(1 + |\vec{v}|^2) = \text{L.H.S.}$$

67.  $(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})$

$$= \vec{0} + \vec{B} \times \vec{A} + \vec{A} \times \vec{C} + \vec{B} \times \vec{C} \quad [\because \vec{A} \times \vec{A} = \vec{0}]$$

$$= \vec{B} \times \vec{A} + \vec{A} \times \vec{C} + \vec{B} \times \vec{C}$$

Now,  $[(\vec{A} + \vec{B}) \times (\vec{A} + \vec{C})] \times (\vec{B} \times \vec{C})$

$$\begin{aligned}
 &= [\vec{B} \times \vec{A} + \vec{A} \times \vec{C} + \vec{B} \times \vec{C}] \times (\vec{B} \times \vec{C}) \\
 &= (\vec{B} \times \vec{A}) \times (\vec{B} \times \vec{C}) + (\vec{A} \times \vec{C}) \times (\vec{B} \times \vec{C}) \\
 &= \{(\vec{B} \times \vec{A}) \cdot \vec{C}\} \vec{B} - \{(\vec{B} \times \vec{A}) \cdot \vec{B}\} \vec{C} \\
 &\quad + \{(\vec{A} \times \vec{C}) \cdot \vec{C}\} \vec{B} - \{(\vec{A} \times \vec{C}) \cdot \vec{B}\} \vec{C} \\
 &\quad [\because (\vec{a} \times \vec{b}) \times \vec{c} = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{b} \cdot \vec{c})\vec{a}] \\
 &= [\vec{B} \vec{A} \vec{C}] \vec{B} - [\vec{A} \vec{C} \vec{B}] \vec{C}
 \end{aligned}$$

$$\begin{aligned}
 &[\because [\vec{A} \vec{B} \vec{C}] = 0 \text{ if any two of } [\vec{A}, \vec{B}, \vec{C}] \text{ are equal.}] \\
 &= [\vec{A} \vec{C} \vec{B}] \{\vec{B} - \vec{C}\}
 \end{aligned}$$

$$\begin{aligned}
 \text{LHS} &= [\vec{A} \vec{C} \vec{B}] \{(\vec{B} - \vec{C}) \cdot (\vec{B} + \vec{C})\} \\
 &= [\vec{A} \vec{C} \vec{B}] \{|\vec{B}|^2 - |\vec{C}|^2\} = 0 \quad [\because |\vec{B}| = |\vec{C}|]
 \end{aligned}$$

68. Given that  $AD = 4$   
Let  $DE = h$

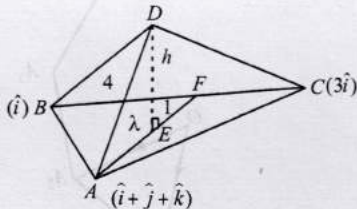
$$\text{Volume of tetrahedron} = \frac{2\sqrt{2}}{3}$$

$$\Rightarrow \frac{1}{3} Ar(\Delta ABC)h = \frac{2\sqrt{2}}{3}$$

$$\therefore \frac{1}{2} |\vec{BA} \times \vec{BC}| h = 2\sqrt{2}$$

$$\frac{1}{2} |(\hat{j} + \hat{k}) \times 2\hat{i}| h = 2\sqrt{2} \text{ or } |\hat{j} - \hat{k}| h = 2\sqrt{2}$$

$$\text{or } \sqrt{2}h = 2\sqrt{2} \therefore h = 2$$



Let point E divides median AF in the ratio  $\lambda : 1$

$$\therefore \vec{OE} = \frac{\lambda \cdot 2\hat{i} + (\hat{i} + \hat{j} + \hat{k})}{\lambda + 1} \quad \dots(ii)$$

$$\therefore \vec{AE} = \text{P.V. of } E - \text{P.V. of } A = \frac{\lambda}{\lambda + 1} (\hat{i} - \hat{j} - \hat{k})$$

$$\therefore |\vec{AE}|^2 = \vec{AE} \cdot \vec{AE} = \left(\frac{\lambda}{\lambda + 1}\right)^2 \cdot 3 \quad \dots(iii)$$

$$\text{Now, } h^2 + AE^2 = AD^2$$

$$\Rightarrow 4 + \left(\frac{\lambda}{\lambda + 1}\right)^2 \cdot 3 = 16 \Rightarrow 3\left(\frac{\lambda}{\lambda + 1}\right)^2 = 12$$

$$\Rightarrow \left(\frac{\lambda}{\lambda + 1}\right) = \pm 2 \Rightarrow \lambda = \pm(2\lambda + 2)$$

$$\therefore \lambda = -2 \text{ or } -2/3$$

Putting the value of  $\lambda$  in (ii) we get the possible positions of E as  $(-1, 3, 3)$  or  $(3, -1, -1)$

69. Given that  $\vec{b}, \vec{c}, \vec{d}$  are not coplanar  $\therefore [\vec{b}, \vec{c}, \vec{d}] \neq 0$

$$\begin{aligned}
 \text{Here, } (\vec{a} \times \vec{b}) \times (\vec{c} \times \vec{d}) &= -(\vec{c} \times \vec{d}) \times (\vec{a} \times \vec{b}) \\
 &= -(\vec{c} \times \vec{d} \cdot \vec{b})\vec{a} + (\vec{c} \times \vec{d} \cdot \vec{a})\vec{b} \\
 &= [\vec{a} \vec{c} \vec{d}]\vec{b} - [\vec{b} \vec{c} \vec{d}]\vec{a} \quad \dots(i)
 \end{aligned}$$

$$\begin{aligned}
 (\vec{a} \times \vec{c}) \times (\vec{d} \times \vec{b}) &= -(\vec{d} \times \vec{b}) \times (\vec{a} \times \vec{c}) \\
 &= -(\vec{d} \times \vec{b} \cdot \vec{c})\vec{a} + (\vec{d} \times \vec{b} \cdot \vec{a})\vec{c} \\
 &= [\vec{a} \vec{d} \vec{b}]\vec{c} - [\vec{c} \vec{d} \vec{b}]\vec{a} \quad \dots(ii)
 \end{aligned}$$

$$\begin{aligned}
 (\vec{a} \times \vec{d}) \times (\vec{b} \times \vec{c}) &= (\vec{a} \times \vec{d}) \cdot \vec{c} \vec{b} - (\vec{a} \times \vec{d} \cdot \vec{b})\vec{c} \\
 &= -[\vec{a} \vec{c} \vec{d}]\vec{b} - [\vec{a} \vec{d} \vec{b}]\vec{c} \quad \dots(iii)
 \end{aligned}$$

Adding (i), (ii) and (iii), we get.

$$\text{given vector} = -2[\vec{b} \vec{c} \vec{d}]\vec{a} = k\vec{a}$$

$\Rightarrow$  given vector is parallel to  $\vec{a}$ .

70. Let  $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\text{Then } \vec{R} \times \vec{B} = \vec{C} \times \vec{B}$$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ 1 & 1 & 1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -3 & 7 \\ 1 & 1 & 1 \end{vmatrix}$$

$$\Rightarrow (y - z)\hat{i} - (x - z)\hat{j} + (x - y)\hat{k} = -10\hat{i} + 3\hat{j} + 7\hat{k}$$

$$\Rightarrow y - z = -10 \quad \dots(i)$$

$$z - x = +3 \quad \dots(ii)$$

$$x - y = 7 \quad \dots(iii)$$

$$\text{Also } \vec{R} \cdot \vec{A} = 0$$

$$\Rightarrow 2x + z = 0 \quad \dots(iv)$$

From (ii) and (iv) we get  $x = -1$ , from (i) and (iii) we get.

$$\Rightarrow y = -8 \text{ and } z = 2$$

$$\therefore \vec{R} = -\hat{i} - 8\hat{j} + 2\hat{k}$$

71. Given that  $\vec{a}, \vec{b}, \vec{c}$  are three coplanar vectors.

$\therefore$  There exists scalars  $x, y, z$ , not all zero, such that

$$x\vec{a} + y\vec{b} + z\vec{c} = \vec{0} \quad \dots(i)$$

Taking dot product of eqn. (i) with  $\vec{a}$  and  $\vec{b}$  respectively, we get

$$x\vec{a} \cdot \vec{a} + y\vec{a} \cdot \vec{b} + z\vec{a} \cdot \vec{c} = 0 \quad \dots(ii)$$

$$x\vec{b} \cdot \vec{a} + y\vec{b} \cdot \vec{b} + z\vec{b} \cdot \vec{c} = 0 \quad \dots(iii)$$

Since equations (i), (ii), (iii) form a homogeneous system of equations, where  $x, y, z$  are not all zero. Therefore system must have non trivial solution.



$$\therefore \begin{vmatrix} \vec{a} & \vec{b} & \vec{c} \\ \vec{a}\vec{a} & \vec{a}\vec{b} & \vec{a}\vec{c} \\ \vec{b}\vec{a} & \vec{b}\vec{b} & \vec{b}\vec{c} \end{vmatrix} = 0 \quad \text{Hence Proved.}$$

72. Let the position vectors of points  $A, B, C, D$  with respect to origin  $O$  be  $a, b, c,$  and  $d$  respectively.

$$\begin{aligned} \text{Then, } \vec{AB} &= \vec{b} - \vec{a}, & \vec{AD} &= \vec{d} - \vec{a}, \\ \vec{BC} &= \vec{c} - \vec{b}, & \vec{BD} &= \vec{d} - \vec{b}, \\ \vec{CD} &= \vec{d} - \vec{c}, & \vec{CA} &= \vec{a} - \vec{c} \end{aligned}$$

$$\begin{aligned} \text{Now, } |\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}| \\ = |(\vec{b} - \vec{a}) \times (\vec{d} - \vec{c}) + (\vec{c} - \vec{b}) \times (\vec{d} - \vec{a}) + (\vec{a} - \vec{c}) \times (\vec{d} - \vec{b})| \\ = |\vec{b} \times \vec{d} - \vec{a} \times \vec{d} - \vec{b} \times \vec{c} + \vec{a} \times \vec{c} + \vec{c} \times \vec{d} - \vec{c} \times \vec{a} \\ - \vec{b} \times \vec{d} + \vec{b} \times \vec{a} + \vec{a} \times \vec{d} - \vec{a} \times \vec{b} - \vec{c} \times \vec{d} + \vec{c} \times \vec{b}| \\ = |-\vec{b} \times \vec{c} + \vec{a} \times \vec{c} - \vec{c} \times \vec{a} + \vec{b} \times \vec{a} - \vec{a} \times \vec{b} + \vec{c} \times \vec{b}| \\ = 2|\vec{b} \times \vec{a} + \vec{c} \times \vec{b} + \vec{a} \times \vec{c}| \quad \dots(i) \end{aligned}$$

Also Area of  $\Delta ABC$  is

$$\begin{aligned} &= \frac{1}{2} |\vec{BC} \times \vec{BA}| = \frac{1}{2} |(\vec{c} - \vec{b}) \times (\vec{a} - \vec{b})| \\ &= \frac{1}{2} |(\vec{c} \times \vec{a} - \vec{c} \times \vec{b} - \vec{b} \times \vec{a} + \vec{b} \times \vec{b})| \\ &= \frac{1}{2} |-\vec{b} \times \vec{a} - \vec{c} \times \vec{b} - \vec{a} \times \vec{c}| = \frac{1}{2} |\vec{b} \times \vec{a} + \vec{c} \times \vec{b} + \vec{a} \times \vec{c}| \\ &\Rightarrow 2Ar(\Delta ABC) = |\vec{b} \times \vec{a} + \vec{c} \times \vec{b} + \vec{a} \times \vec{c}| \quad \dots(ii) \end{aligned}$$

From (i) and (ii), we get

$$\begin{aligned} |\vec{AB} \times \vec{CD} + \vec{BC} \times \vec{AD} + \vec{CA} \times \vec{BD}| \\ = 2(2Ar(\Delta ABC)) = 4Ar(\Delta ABC) \quad \text{Hence Proved.} \end{aligned}$$

73. From given position vector  $\vec{AB} = -\hat{i} + 5\hat{j} - 3\hat{k},$

$$\vec{AC} = -4\hat{i} + 3\hat{j} + 3\hat{k}$$

$$\vec{AD} = \hat{i} + 7\hat{j} + (1 + \lambda)\hat{k}$$

Given that  $A, B, C, D$  lie in a plane then  $\vec{AB}, \vec{AC}, \vec{AD}$  are coplanar i.e.  $[\vec{AB} \vec{AC} \vec{AD}] = 0$

$$\Rightarrow \begin{vmatrix} -1 & 5 & -3 \\ -4 & 3 & 3 \\ 1 & 7 & 1 + \lambda \end{vmatrix} = 0$$

$$\Rightarrow -1(3 + 3\lambda - 21) - 5(-4 - 4\lambda - 3) - 3(-28 - 3) = 0$$

$$\Rightarrow -3\lambda + 18 + 20\lambda + 35 + 93 = 0 \Rightarrow 17\lambda + 146 = 0$$

$$\Rightarrow \lambda = -\frac{146}{17}$$

74.  $\vec{r} = \lambda x\hat{i} + \lambda y\hat{j} + \lambda z\hat{k}$

On comparing both sides, we get

$$\Rightarrow x + 3y - 4z = \lambda x \Rightarrow (1 - \lambda)x + 3y - 4z = 0$$

$$\Rightarrow x - 3y + 5z = \lambda y \Rightarrow x - (3 + \lambda)y + 5z = 0$$

$$\Rightarrow 3x + y + 0z = \lambda z \Rightarrow 3x + y - \lambda z = 0$$

Since  $x, y, z \neq (0, 0, 0)$  then all the above three equations non zero solution.

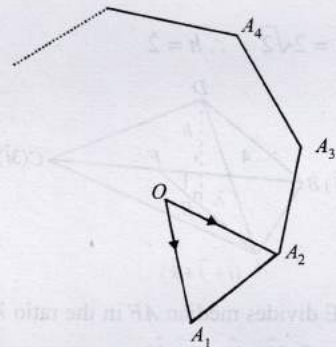
$$\therefore \begin{vmatrix} 1 - \lambda & 3 & -4 \\ 1 & -(3 + \lambda) & 5 \\ 3 & 1 & -\lambda \end{vmatrix} = 0$$

$$\Rightarrow (1 - \lambda)[3\lambda + \lambda^2 - 5] - 3[-\lambda - 15] - 4[1 + 9 + 3\lambda] = 0$$

$$\Rightarrow \lambda^3 + 2\lambda^2 + \lambda = 0 \Rightarrow \lambda(\lambda + 1)^2 = 0 \Rightarrow \lambda = 0, -1.$$

75. Since  $A_1, A_2, \dots, A_n$  are the vertices of a regular plane polygon.

$\therefore \vec{OA}_1, \vec{OA}_2, \dots, \vec{OA}_n$  all vectors are of same magnitude, say 'a' and angle between any two consecutive vector is same that is  $\frac{2\pi}{n}$  radians. Let  $\hat{n}$  be the normal unit vectors perpendicular to the plane of the polygon.



$$\therefore \vec{OA}_1 \times \vec{OA}_2 = a^2 \sin \frac{2\pi}{n} \hat{n} \quad \dots(i)$$

$$\begin{aligned} \text{Now, } \sum_{i=1}^{n-1} \vec{OA}_i \times \vec{OA}_{i+1} &= \sum_{i=1}^{n-1} a^2 \sin \frac{2\pi}{n} \hat{n} \\ &= (n-1)a^2 \sin \frac{2\pi}{n} \hat{n} = -(n-1)[\vec{OA}_2 \times \vec{OA}_1] \\ & \quad \text{[using eq}^n \text{ (i)]} \\ &= (1-n)[\vec{OA}_2 \times \vec{OA}_1] = R.H.S \end{aligned}$$